Metrics-based Software Vulnerability Discovery Model to Support Cybersecurity Test & Evaluation

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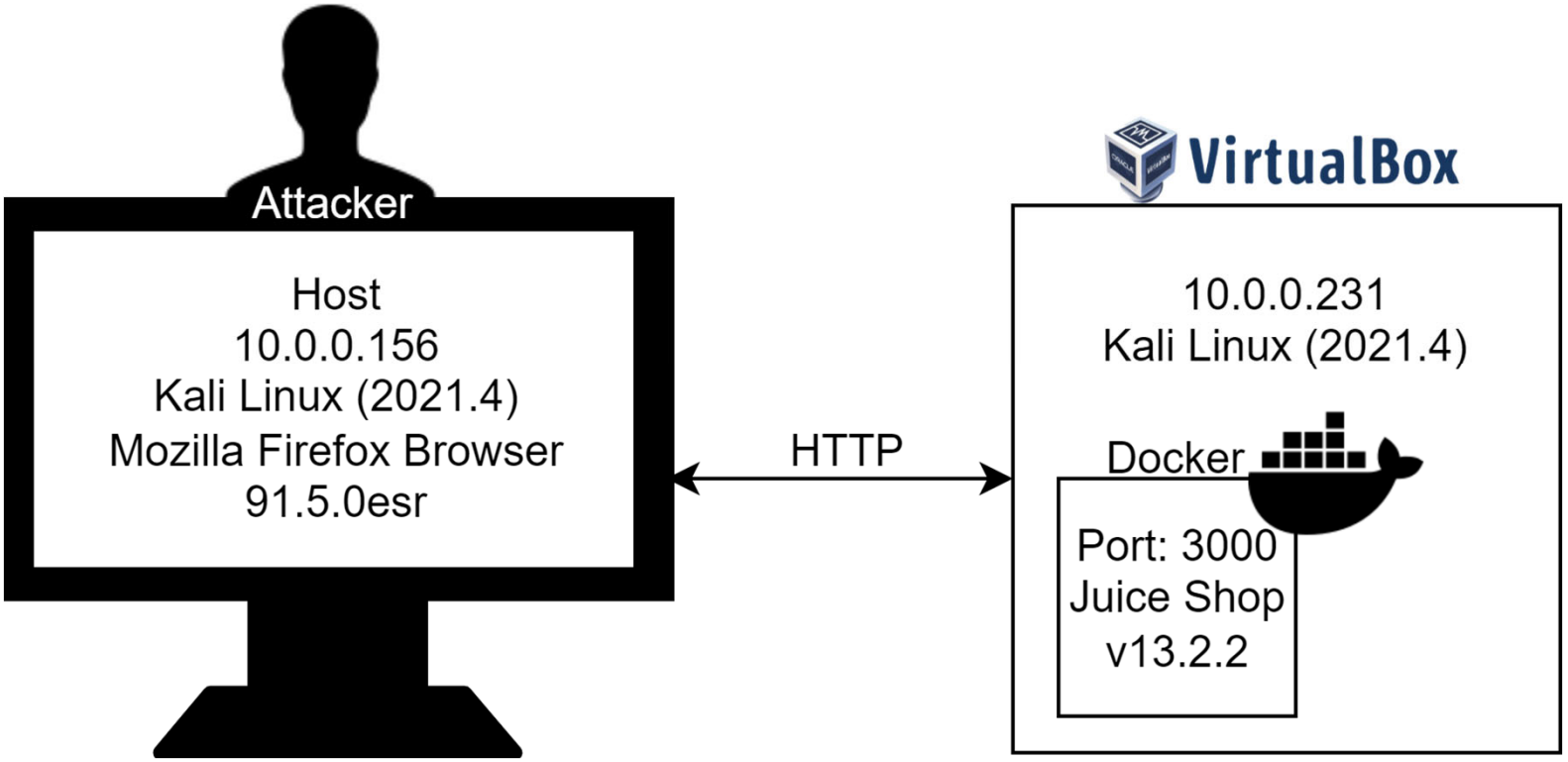
Background and Introduction

- Software highly versatile
  - Provides wide variety of functionality in applications and software-enabled systems
- Potentially negative impacts of software exploitation cast dark shadow over otherwise promising nature of software
  - Monitor and control infrastructure, homeland defense, and national security
- Effort dedicated to vulnerability taxonomies, techniques, and tools
- Some attention to vulnerability discovery models (VDM)
- Only consider amount of time spent testing
  - Prevents more detailed guidance on relative effectiveness of alternative vulnerability testing tools and techniques

Contributions

- Study comparing software VDM incorporating covariates and Alhazmi-Malaiya Logistic (AML) model
  - Covariate VDM links specific test activities and tools to vulnerability discovery, organizations
  - AML flexible and highly cited VDM without covariates
- Target application identified and subjected to multiple vulnerability discovery tools and techniques
- Software VDM incorporating covariates
  - More accurately tracked and predicted number of vulnerabilities discovered in future intervals as function of penetration testing activities performed
  - Achieved significantly better goodness of fit, despite information theoretic measures penalized covariate models for additional parameters

Environment Architecture Setup



- Application emulates modern day web application for an online Juice Shop marketplace with complex functionality, containing a wide variety of vulnerabilities

- A01: Broken Access Control
- A02: Cryptographic Failures
- A03: Injection
- A04: Insecure Design
- A05: Security Misconfiguration
- A06: Vulnerable and Outdated Components
- A07: Identification and Authentication Failures
- A08: Software and Data Integrity Failures
- A09: Security Logging and Monitoring Failures
- A10: Server-Side Request Forgery

Data Collection

TABLE II
DATA COLLECTED DURING PENETRATION TESTING ACTIVITIES

T	K	OWASP Top 10										Tools		
		A01	*A02*	*A03*	*A04*	*A05*	*A06*	*A07*	*A08*	*A09*	*A10*	Z	B	M
1	4	2	0	0	1	1	0	0	0	0	0	147	42	0
2	12	2	1	3	1	1	1	2	0	1	0	375	402	104
3	8	1	1	2	0	0	0	3	0	1	0	0	521	195
4	6	4	1	0	1	0	0	0	0	0	0	11	123	34
5	3	0	1	1	1	0	0	1	0	0	0	0	45	6
6	4	2	0	0	1	0	0	0	1	0	0	0	71	97
7	3	1	0	1	0	0	0	1	0	0	0	0	214	0
8	3	1	0	1	0	0	0	0	0	0	1	0	86	107

- **T** – vector of discrete time interval (approximately 5 hours each)
- **K** – vector of number of vulnerabilities discovered in each time interval
- ***A##*** – Number of OWASP Top 10 category vulnerabilities discovered
- **Z**, **B**, **M** – vector of seconds spent executing tools (Zap, Burp, or manual inspection)

Discrete Cox Proportional Hazards NHPP SRGM

- MVF predicts number of vulnerabilities discovered up to and including n^{th} interval

$$m(x) = \omega \sum_{i=1}^{n} p_{i,x_i} \tag{1}$$

  - Given covariates x, where $\omega > 0$ denotes number of vulnerabilities that would be discovered with infinite testing
- Probability that vulnerability discovered in interval i, given it was not discovered in first $(i - 1)$ intervals

$$p_{i,x_i} = \left(1 - \left(1 - h(i)\right)^{g(x_i;\beta)}\right) \prod_{k=1}^{i-1} \left(1 - h(k)\right)^{g(x_k;\beta)} \tag{2}$$

  - $h(\cdot)$ - baseline hazard function
  - β - vector of covariate coefficient within Cox proportional hazards model

$$g(x_i;\beta) = \exp(\beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_m x_{im}) \tag{3}$$

Hazard Functions

- Can be incorporated into Equation (2)
  - Geometric (GM):

$$h(b) = b \tag{4}$$

    - where $b \in (0,1)$
  - Second order Negative binomial (NB2):

$$h(i;b) = \frac{ib^2}{1 + b(i-1)} \tag{5}$$

    - where $b \in (0,1)$ and 2 indicates order
  - Second order Discrete Weibull (DW2):

$$h(i;b) = 1 - b^{i^2-(i-1)^2} \tag{6}$$

    - where $b \in (0,1)$ and 2 indicates order

Log-Likelihood Function

- LL function of discrete Cox proportional hazard NHPP SRGM

$$LL(\boldsymbol{\gamma}, \boldsymbol{\beta}, \omega) = -\omega \sum_{i=1}^{n} p_{i,x_i} + \sum_{i=1}^{n} y_i \ln(\omega) + \sum_{i=1}^{n} y_i \ln\left(p_{i,x_i}\right) - \sum_{i=1}^{n} \ln(y_i!) \quad (7)$$

  - $\boldsymbol{\gamma}$ is vector of hazard function parameters
  - $\boldsymbol{\beta}$ is vector of m covariate coefficients
  - ω is the total number of vulnerabilities to be discovered
  - $\boldsymbol{y}_i$ is the number of defects discovered in the i^{th} interval
  - $\boldsymbol{y}_n$ is vector of vulnerabilities discovered in each of the n intervals
  - $\boldsymbol{x}$ is the given covariate data

Log-Likelihood Function (2)

$$\hat{\omega} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} p_{i,x_i}} \tag{8}$$

- Substitute for $\hat{\omega}$ in LL function to obtain reduced log-likelihood (RLL) function

$$\frac{\partial RLL}{\partial \boldsymbol{\beta}} = 0 \tag{9}$$

And

$$\frac{\partial RLL}{\partial \boldsymbol{\gamma}} = 0 \tag{10}$$

Alhazmi-Malaiya Logistic (AML) Model

- Mean Value Function

$$m(t) = \frac{B}{Bce^{-ABt} + 1} \qquad (11)$$

  - B - number of vulnerabilities that would be discovered with indefinite testing
  - A and c - constant of proportionality characterizing vulnerability discovery rate

- Vulnerability Discovery Intensity Function

$$\lambda(t) = \frac{B^3 cAe^{-ABt}}{(Bce^{-ABt}+1)^2} \qquad (12)$$

Alhazmi-Malaiya Logistic (AML) Model (2)

- Likelihood Function

$$LL(\mathbf{T},\mathbf{K};\theta) = \sum_{i=1}^{n} k_i \log(m(t_i) - m(t_{i-1}) - \sum_{i=1}^{n} \log(k_i!) - \sum_{i=1}^{n} (m(t_i) - m(t_{i-1})) \quad (13)$$

  - Where $<\mathbf{T},\mathbf{K}>=<(t_1, k_1), (t_2, k_2),\ldots,(t_n, k_n)>$
  - t_i is the time at which the i^{th} interval ended
  - k_i is the number of vulnerabilities discovered in interval i
  - $\theta = \{A, B, c\}$ is the vector of model parameters

$$\frac{\partial LL}{\partial \theta} = 0 \quad (14)$$

Model Assessment

- Sum of Square Error (SSE):

$$SSE = \sum_{i=1}^{n} \left(N(i) - \hat{m}(i) \right)^2 \qquad (15)$$

- Predictive Sum of Square Error (PSSE):

$$PSSE = \sum_{i=(n-l+1)}^{n} \left(N(i) - \hat{m}(i) \right)^2 \qquad (16)$$

  - $N(i)$ - vector of vulnerabilities discovered by time t_i
  - $\hat{m}(t_i)$- fitted model's estimate of number of vulnerabilities discovered

Model Assessment (2)

- Akaike Information Criterion (*AIC*):

$$AIC = 2\nu - 2LL(\hat{\gamma}) \quad (17)$$

  - ν - number of model parameters
  - Penalizes model by 2 points for each parameter

- Bayesian Information Criterion *(BIC)*:

$$BIC = \nu \log(n) - 2LL(\hat{\gamma}) \quad (18)$$

  - Penalty size includes the sample size (n)

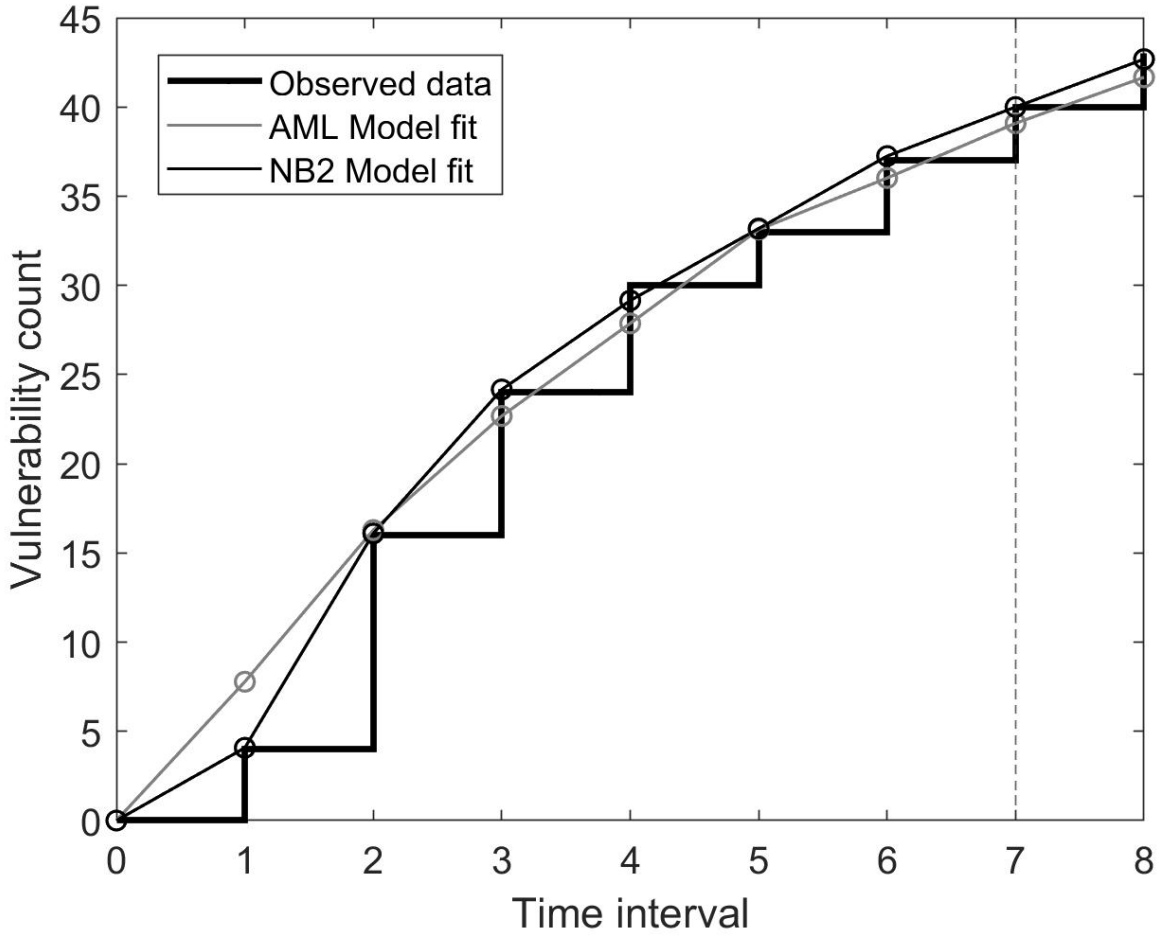
Goodness of Fit Model Assessment

TABLE III
GOODNESS OF FIT VULNERABILITY DISCOVERY MODELS WITH AND WITHOUT COVARIATES

Model	SSE	$PSSE$	AIC	BIC
AML	34.2907	5.6025	47.0145	47.2528
AML ($Z_i + B_i + M_i$)	21.5592	1.7770	41.8812	42.1195
GM	6.6994	2.7028	35.5228	35.9200
NB2	0.9573	0.1007	**34.9565**	**35.3537**
DW2	13.8840	2.8550	40.8420	41.2390
DW3	1.2286	0.3291	36.9901	37.4667
S	**0.8891**	**0.0857**	36.9387	37.4153
TL	6.4349	2.5106	37.5281	38.0047
IFRSB	95.1872	8.9720	65.7081	66.1053
IFRGSB	6.4503	2.5154	37.5306	38.0072

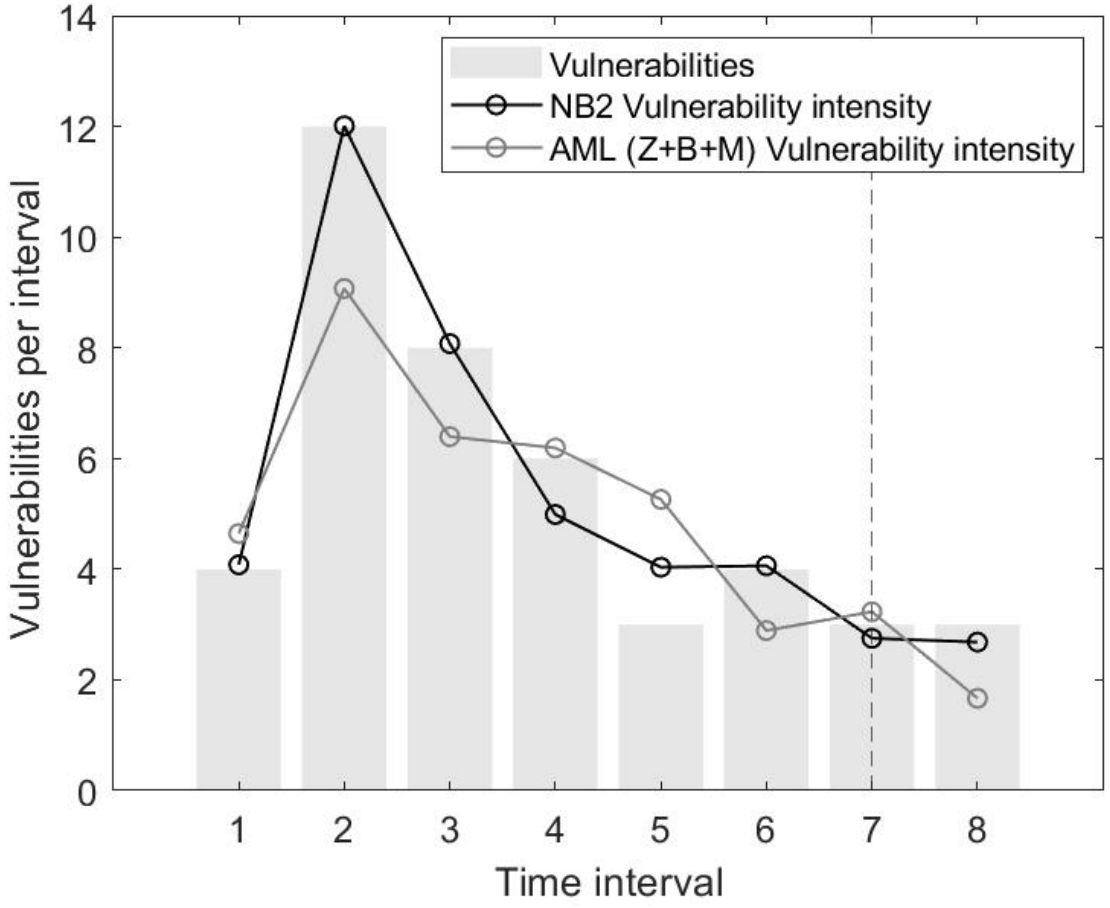
- VDM with covariates performed substantially better, despite AIC and BIC measures penalized inclusion of additional parameters for covariates associated with vulnerability discovery activities

Model Fit



- Covariate VDM with NB2 hazard function tracked and predicted better than AML model

Vulnerability Discovery Intensity



- Covariate VDM with NB2 hazard function tracked and predicted better than AML model

Summary & Conclusion

- Presented comparative study of software VDM incorporating covariates with Alhazmi-Malaiya Logistic (AML) model
- Software VDM incorporating covariates
  - More accurately tracked and predicted number of vulnerabilities discovered in future intervals as function of penetration testing activities performed
  - Achieved significantly better goodness of fit, despite information theoretic measures penalized covariate models for additional parameters
- Open source tool available from
  - https://lfiondella.sites.umassd.edu/research/software-reliability/

Next Steps and Future Work

- Combine techniques presented here with other reliability engineering techniques to provide more comprehensive, accurate, and usable methods that support systematic test and evaluation of software
- Demonstrating applicability of models to test & evaluation of machine learning with domain specific techniques and tools such as adversarial machine learning, generative adversarial training, and incremental learning

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