EFFICIENT MODELING & SIMULATION USING DESIGN OF EXPERIMENTS

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OUTLINE

• Background & Resources
• Why Use DOE for M&S?
• Why is DOE important?
• Overview of Design of Experiments (DOE)
• Efficient M&S Using DOE – 3 Examples
  • Sequential traditional DOE
  • Space-Filling DOE Case Study
  • Sequential space-filling DOE
USING DESIGN OF EXPERIMENTS (DOE) FOR 35 YEARS

- ‘83-’87 Honeywell, Inc., Engineer
  First saw the power of DOE in 1984
- ‘87-’99 ECHIP, Inc., Partner & Technical Director
  200+ DOE courses, on-site at 40+ companies
- ‘99-’05 Peak Process, LLC, Consultant
- ‘05-’08 US Army, Edgewood Chemical Biological Center (ECBC),
  Modeling, Simulation, & Analysis Branch
  DOE with Real data and Modeling & Simulation data
- ‘08-’19 SAS Institute Inc., JMP Division
  Data Visualization, Data Analytics, and their synergy with DOE
  Support DoD sites, NASA, & Defense Contractors
PROJECTS USING DOE AT U.S. ARMY ECBC CY05-08

Detection, Decontamination & Protection

- JPM Nuclear Biological Chemical Contamination Avoidance (NBCCA) - Whole Systems Live Agent Test (WSLAT) Team support to the Joint Biological Point Detection System (JBPDS)
- Agent Fate wind tunnel experiments
- Decontamination Sciences Team
  - Contact Hazard Residual Hazard Efficacy Agent T&E Integrated Variable Environment (CREATIVE) - real and simulation data
  - Modified vaporous hydrogen peroxide (mVHP) decontamination – real data
- Smoke and Target Defeat Team
  - Pepper spray characterization – real data
  - Obscurant material evaluation (with OptiMetrics, Inc.) – simulation data
- U.S. Army Independent Laboratory In-house Research (ILIR) on novel DOE used with simulations
  - **Re-analysis of USAF Kunsan AFB Focused Effort BWA simulation data**
  - CB Sim Suite used for sensitivity analysis of atmospheric stability
- U.S. Marine Corps Expeditionary Biological Detection (EBD) Advanced Technology Demonstration (ATD)
  - Chamber testing of detectors – real data
  - CB Sim Suite sensor deployment studies – simulation data
- U.S. Navy lead on Joint Expeditionary Collective Protection (JECP)
  - Swatch and chamber testing – real data
  - **Computational Fluid Dynamics (CFD) – simulation data**
DOWNLOADS

- PDFs available
  - Dissertation 2017 - A framework for the optimization of doctrine and systems in Army Air Defense units using predictive models of stochastic computer simulations – LTC Brian Wade, Technical Director at TRAC MRY
    https://smartechgatechedu/handle/1853/58275
These 12 videos primarily cover Design of Experiments (DOE) topics.

<table>
<thead>
<tr>
<th>Custom DOE - JMP 13 (not 14)</th>
<th>Screening Designs</th>
<th>Compare Designs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make the Design Fit Your Problem (Link to Mastering JMP)</td>
<td>Classic FF &amp; PB, and Modern D-Optimal, Supersaturated, DSD, &amp; Alias-Optimal</td>
<td>How to Choose Better Designs on Multiple Criteria</td>
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<table>
<thead>
<tr>
<th>Advanced Custom DOE - JMP 13 (not 14)</th>
<th>Definitive Screening Designs (DSD)</th>
<th>Data Transformations</th>
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<tbody>
<tr>
<td>Augmentation, Broken Design Repair, &amp; Design from a Candidate Set</td>
<td>Creation &amp; Augmentation</td>
<td>Get Rid of L-o-F, Predictions Make Physical Sense (Link to Mastering JMP)</td>
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<table>
<thead>
<tr>
<th>Mixture DOE</th>
<th>Analyzing DSD DOEs</th>
<th>Power Calculation via MC Simulation</th>
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<tbody>
<tr>
<td>Efficiently Blending Ingredients to Optimize a Process (Link to Mastering JMP)</td>
<td>Graphical Methods and Fit Definitive Screening Platform</td>
<td>Binary Responses &amp; Split-Plot Designs</td>
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<table>
<thead>
<tr>
<th>Efficient M&amp;S Using DOE</th>
<th>Exploratory Data and Root Cause Analyses</th>
<th>Covering Arrays - Rapid Fault Detection in Software &amp; Systems</th>
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<tbody>
<tr>
<td>How to Run Fewer Computer Simulations</td>
<td>What to Do When You Don’t Have a DOE</td>
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</tr>
</tbody>
</table>

1.6 Comparative Assessment and Decision Support System for Strategic Military Airlift Capability

Comparative Assessment and Decision Support System for Strategic Military Airlift Capability

John Salmon, Curtis Iwata, Dimitri Mavris and Neil Weston
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ABSTRACT

The Lockheed Martin Aeronautics Company has been awarded several programs to modernize the aging C-5 military transport fleet. In order to ensure its continuation amidst budget cuts, it was important to engage the decision makers by providing an environment to analyze the benefits of the modernization program. This paper describes an interface that allows the user to change inputs such as the scenario airfields, take-off conditions, and reliability characteristics. The underlying logistics surrogate model was generated using data from a discrete-event simulation. Various visualizations, such as intercontinental flight paths illustrated in 3D, have been created to aid the user in analyzing scenarios and performing comparative assessments for various output logistics metrics. The capability to rapidly and dynamically evaluate and compare scenarios was developed enabling real-time strategy exploration and trade-offs.

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WHY USE DESIGN OF EXPERIMENTS METHODS WITH SIMULATION EXPERIMENTS?

*Quicker answers, lower costs, solve bigger problems*

- Obtain a fast surrogate model of the simulation
  - Individual simulations can run for hours, days, weeks
    - Computational Fluid Dynamics (CFD) or Simulation runs in real-time
  - Numbers of factors can be very large (100+)
  - Numbers of simulations needed can be large (thousands in many cases)
  - Simulations can be stochastic requiring many replications
- Surrogate model yields a fast approximation of the simulation
  - more rapidly answer “what if?” questions – *Instantaneous answer for any “NEW” scenario!*
  - do sensitivity analysis of the control factors
  - optimize multiple responses and make trade-offs
- By running sequences of designs one can be as *cost effective as possible* & *run no more trials than are needed* to get a useful answer
- By running efficient subsets of all possible combinations, one can – for the same resources and constraints – *solve bigger problems*
WHY IS USING DOE IMPORTANT?

• “One thing we have known for many months is that the spigot of defense funding opened by 9/11 is closing.”
• “In the past, modernization programs have sought a 99 percent solution over a period of years, rather than a 75 percent solution over a period of weeks or months.”
  • Two quotes from the January 27, 2009 submitted statement of Secretary of Defense Robert M. Gates to the Senate Armed Services Committee.

• DOE is one of the more powerful tools we can use to efficiently accomplish our goals.
  • DOE yields the maximum information from the fewest experiments.
  • DOE often yields an 80% solution in less than 20% of the work.
**LONG RUNNING PHYSICS-BASED SIMULATIONS**

Detailed Physics Models can require a great deal of runtime to generate a short period of simulation time.

<table>
<thead>
<tr>
<th>Computational Fluid Dynamics (CFD) Models</th>
<th>Lagrangian-Particle</th>
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<tbody>
<tr>
<td>Developed for Interior Moving Man in Simulation</td>
<td>Developed for Exterior Stationary Grids</td>
</tr>
<tr>
<td>8M cells</td>
<td>TBD Cells</td>
</tr>
<tr>
<td>10 Seconds of Simulation</td>
<td>External CW Deposition/ Evaporation, Vegetation, Solar Heating</td>
</tr>
<tr>
<td>64 CPUs – 4K slower</td>
<td></td>
</tr>
<tr>
<td>12 Hours of Runtime</td>
<td></td>
</tr>
<tr>
<td>Detailed Ingress/Egress, Internal Airflow and Convection</td>
<td></td>
</tr>
</tbody>
</table>

- Developed for Exterior Stationary Grids
- 1.5M Cells
- 30 Seconds of Simulation
- Single CPU – 20K slower
- 7 Days of Runtime

- Speed, Flexibility, More User Friendly, V&V

**Speed, Flexibility, More User Friendly, V&V**
STOCHASTIC SIMULATIONS WITH MANY REPLICATES

Agent Based Simulations
STOCHASTIC SIMULATIONS WITH MANY REPLICATES

Discrete Event Simulations
CLASSIC DEFINITION OF DOE

- Purposeful control of the inputs (factors) in such a way as to deduce their relationships (if any) with the output (responses).
**RESPONSE SURFACE DOE IN A NUTSHELL**

- **Fit requires data from all 3 blocks**
- **Can fit data from blocks 1, 2 or 3**
- **Fit requires data from blocks 1 & 2**
- **Fit requires data from all 3 blocks**
4 CONTROLS (INPUTS) & 2 RESPONSES (OUTPUTS) AND THEIR EMPIRICAL RELATIONSHIPS (MODEL)

Get these Response Surfaces and Prediction Profiler as result of analyzing data collected for a DOE
For non-stochastic simulations for which a surrogate model has been created, Monte Carlo simulations can be run using assumed distributions for inputs to better assess transmitted variation about the model point estimate.
TWO CLASSES OF DESIGNS FOR TWO TYPES OF SURROGATE MODELING OF SIMULATIONS

- **Traditional factorial/response surface** designs for polynomial modeling with categorical (qualitative) and continuous (quantitative) variables
  - Designs can be sequentially constructed to support increasingly complex models
  - Example featured here reanalyzes a simulation case matrix in which all combinations of 6 variable settings were originally run - a total of $648 = 6 \times 3 \times 3 \times 3 \times 2 \times 2$
  - References on Resolution V, Fractional-Factorial Designs for many (40+) factors

- **Space-filling** designs primarily for use with continuous and categorical variables AND non-stochastic/deterministic responses
  - These designs can support “Gaussian Process” or “Kriging” spatial regression analysis – an interpolation technique, as well as linear regression – an approximation method
HOW ARE SPACE-FILLING DESIGNS DIFFERENT FROM TRADITIONAL DESIGNS?

Response-Surface Design for 3-Variables with 15 Unique Trials

Space-Filling Design for 3 Variables with 17 Unique Trials

Rather than emphasizing high leverage trials ("corners") for a simple polynomial model, space-filling designs "spread" their trials more uniformly through the space to better capture the local complexities of the simulation model.
TRADITIONAL DESIGNS FOR POLYNOMIAL MODELING

• I used to say “If a “textbook” fractional-factorial, orthogonal array or response-surface design is available, then use it.”
  
  Now I say, “If Definitive Screening design is available, then use it.”

• Textbooks and web site catalogs do not always contain designs for categorical variables with:
  • all combinations of mixed numbers of levels (e.g. 3, 4, 5, and 21)
  • large numbers of levels for variables (e.g. 5+)

• Algebraic (Orthogonal Array) and algorithmic (D-optimal) computer generated designs can often be used
  • Orthogonal Arrays (and Nearly Orthogonal Arrays) are good at yielding analysis with unconfounded estimates of the “main effects” when variables have many different levels
  • D-optimal designs are good for adding on the fewest additional trials to support higher order “interaction” terms in the model
SEQUENTIAL DESIGNS

• Simulation experiments – Sequential designs are easily employed because “restricted randomization” is not an issue
  • Many simulations are deterministic
  • Even if stochastic (random), correlation with unknown factors is not possible
  • All factors are generally just as easy to change
  • Can still inexpensively add a blocking variable to test if “the code has been changed!”

• Real experiments – The issue of “restricted randomization” does arise making sequential experimentation a bit more complicated – but still possible to employ
  • Groups of trials run at different (even widely spaced) periods of time
    • Addressed using a blocking factor
  • Sometimes there are factors that are harder to change than others, e.g. Oven Temperature
    • Addressed using split-plot designs
CASE MATRIX AS USED IN STUDY OF THE OBSERVED RESPONSE “PROBABILITY OF CASUALTY” (PCAS)

<table>
<thead>
<tr>
<th>Variable</th>
<th># Levels</th>
<th>Levels</th>
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</thead>
<tbody>
<tr>
<td>Agent Codes (X1)</td>
<td>6</td>
<td>A, N, T, H, R, Y (categorical)</td>
</tr>
<tr>
<td>Season</td>
<td>3</td>
<td>Winter, Summer, Spring/Fall (categorical)</td>
</tr>
<tr>
<td>Time of Attack (Hour)</td>
<td>3</td>
<td>0500, 1200, 2200 Local Time (continuous)</td>
</tr>
<tr>
<td>No. of TBMs &amp; Spread Radius (X2)</td>
<td>2</td>
<td>1 TBM &amp; 1 m, 2 TBMs &amp; 1000 m (categorical)</td>
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<tr>
<td>Mass (relative)</td>
<td>3</td>
<td>1.00, 1.57, 2.00 (continuous)</td>
</tr>
<tr>
<td>Height of Burst (X3)</td>
<td>2</td>
<td>0, 10 m (continuous)</td>
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<tr>
<td>Total Cases</td>
<td>648</td>
<td></td>
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</table>
ALL 648 POSSIBLE COMBINATIONS OF SETTINGS FOR 6 VARIABLES (6 X 2 X 2 X 3 X 3 X 3)
FOUR STAGE DESIGN SEQUENCE

Stage 1
36 Total Simulations
Design 1, 36 trials

Main effects only for ALL variables + some 2-way interactions
5.6% of 648
324 trials in Design 4 used as checkpoints for Designs 1, 2 & 3

Stage 2
108 Total Simulations
Design 1, 36 trials
Design 2, 72 trials

Stage 1 effects plus all 2-way interactions + some 3-way interactions
16.7% of 648

Stage 3
324 Total Simulations
Design 1, 36 trials
Design 2, 72 trials
Design 3, 216 trials

Stage 2 effects plus all 3-way interactions
50% of 648

Stage 4
ALL 648 Simulations
Design 1, 36 trials
Design 2, 72 trials
Design 3, 216 trials
Design 4, 324 trials

Stage 3 effects plus ALL remaining 4-way, 5-way and 6-way interactions

NOTE: Length of this green box should be longer than shown

Design 4, 324 trials
36 OF ALL 648 POSSIBLE COMBINATIONS OF SETTINGS FOR 6 VARIABLES (6 X 2 X 2 X 3 X 3 X 3)

Red Dots Mark the 36 Trials (an Orthogonal Array) Analyzed for Stage 1
Locations of Trials for a 4-variable, 9-trial Orthogonal Array Design

X1 = 1
X1 = 2
X1 = 3
Delete $x_1$ and View Locations of Trials for a 3-Variable OA9 Design

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
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<tr>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

$x_1 = 1$

$x_1 = 2$

$x_1 = 3$
Projection of Trial Locations
for a 3-variable OA9 Design for All Pairs of Variables

All projections have 9 unique trials that can be used to fit a 2-variable quadratic model with 6 terms.
Can Get Designs from Different Sources

- **Textbook**
  - Limited number of catalogued solutions – experimenters frequently change their problem to match available designs
  - Variable settings are in coded units

- **Web sites of designs**
  - Greater number of catalogued solutions – but never all
  - Variable settings are in coded units

- **Custom computer code**
  - Can find solutions for previously un-catalogued cases
  - Variable settings are in coded units (-1, 0, 1)

- **COTS Solution**
  - Textbook and algorithmic code for generating custom designs
  - Variable settings in natural or laboratory units (120, 150, 180)
Predicted Probability of Casualty (PCAS) vs. Mass – with Mass Treated as a Continuous Variable – for 5 Different Models Fit to 3 Sets of Simulation Data

**1-way model w/nesting**
- Model has 24 terms and fit data from 36 simulations

**Reduced 2-way model**
- Model has 36 terms and fit data from 108 simulations

**Reduced 3-way model**
- Model has 178 terms and fit data from 324 simulations

**1-way model w/nesting + some 2-way terms**
- Model has 31 terms and fit data from 36 simulations

**Reduced 2-way model + some 3-way terms**
- Model has 66 terms and fit data from 108 simulations

Five other variables were held constant at these settings:
- Agent = R
- Season = F
- Time = 12
- HOR = 0
- #TBM & Spread Radius = 1

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![Graphs](image-url)
“FACTOR SPARSITY” AND “EFFECT HEREDITY” USED TO ENHANCE MODEL COMPLEXITY

Factor Sparsity states only a few variables will be active in a factorial DOE.

Effect Heredity states significant interactions will only occur if at least one parent is active.

See Wu & Hamada, p. 112
Higher Resolution (100X) Histograms of the “Percent Off Target” that Response Predictions Fell Relative to 324 Checkpoint Observations

ONLY A FRACTION OF ALL POSSIBLE TRIALS MAY BE REQUIRED TO PROVIDE AN ANSWER

Worst Case = -0.0251%
Half of Cases < 0.0010%

Worst Case = -0.0081%
Half of Cases < 0.0007%

How far off is good enough?
CONCLUSIONS FOR SEQUENTIAL TRADITIONAL DESIGNS

• Possible to get the 80% to 95% solution with less than 20% of the brute force running of all factor combinations
• Use of “factor sparsity” and “effect heredity” principles can help to get more information than the design was originally built to support
• Next stage trials can first be used as checkpoints for previous stages
• With improved efficiency over running all combinations, more factors can be studied with the same resources
HOW ARE SPACE-FILLING DESIGNS DIFFERENT FROM TRADITIONAL DESIGNS?

Rather than emphasizing high leverage trials ("corners") for a simple polynomial model, space-filling designs "spread" their trials more uniformly through the space to better capture the local complexities of the simulation model.
29 CFD SIMULATIONS RUN – 17 USED TO METAMODEL & 12 USED AS CHECKPOINTS

17-trial Orthogonal Latin Hypercube (OLH) space-filling design settings used for creating the metamodel

12-trial Plackett-Burman screening design settings used as checkpoints – half just inside and half just outside design boundary (convex hull)

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<thead>
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<th>Trial</th>
<th>Time of Day</th>
<th>Temperature</th>
<th>Wind Speed</th>
<th>Wind Direction</th>
<th>Relative Humidity</th>
<th>Cloud Cover</th>
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<td>37.5</td>
<td>0.65</td>
</tr>
<tr>
<td>28</td>
<td>972.5</td>
<td>16</td>
<td>3.05</td>
<td>241.875</td>
<td>62.5</td>
<td>0.35</td>
</tr>
<tr>
<td>29</td>
<td>972.5</td>
<td>26</td>
<td>4.55</td>
<td>241.875</td>
<td>37.5</td>
<td>0.35</td>
</tr>
</tbody>
</table>
KRIGING FIT IN 1-D SHOWING INTERPOLATION AND CONFIDENCE INTERVALS ON PREDICTION
SEMINAL PAPER ON “SPACE-FILLING” DOE FOR COMPUTER EXPERIMENTS

- Design and Analysis of Computer Experiments
  Sacks, J., Welch, W.J., Mitchell, T.J. and Wynn, H.P.
  *Statistical Science* 4. 409-423, 1989

- Textbooks on this topic include:
WEBSITES FOR DESIGNS, SOFTWARE & PUBLICATIONS

- http://harvest.nps.edu/ The Simulation Experiments & Efficient Design (SEED) Center for Data Farming at Naval Postgraduate School
  - Designs
    - Nearly Orthogonal Latin Hypercubes (NOLH) and
    - Resolution V, Fractional Factorials for many factors
  - Agent-Based Simulation Software
    - Pythagoras
    - MANA (Map Aware Non-uniform Automata)
  - Many Papers for Download and Links to INFORMS and WSC
- Library of Orthogonal Arrays maintained by Neil J. A. Sloane
  - http://neilsloane.com/oadir/
- Library of Orthogonal Arrays maintained by Warren F. Kuhfield
SURROGATE MODELING OF A COMPUTER SIMULATION
HELICOPTER SURVEILLANCE – IDENTIFYING INSURGENTS

• 2009 International Data Farming Workshop - IDFW21, Lisbon, Portugal
• Largely German team (6 of 8) – their simulation
• 6500 simulations run overnight on cluster in Frankfurt
  • Space Filling Design of Experiments (DOE)
  • 65 unique combinations of 6 factors (each factor at 65 levels)
  • each case had 96 to 100 replications (lost a few)
• Response = Proportion of Insurgents Identified = \( \text{PropIdentINS} \)
  Data bounded between 0 and 1
• Explore data visually first
• **Fit many different models – Regression and Machine Learning**
  using “Train, Validate (Tune), Test” subsets
• Compare Actual vs. Predicted for Test Subsets
SPACE-FILLING DOE (LATIN HYPERCUBE) VISUALIZED WITH 2-D SCATTERPLOT MATRIX AND 3-D SCATTERPLOT
DISTRIBUTIONS OF 1 RESPONSE AND 6 FACTORS

- PropldentINS
- InsurgentCamouflage
- TigerSpeedRelative
- TigerHeight
- Tiger1_Distance
- ConvoySpeed
- num_INS2_AK47
PROPIDENTINS VS. X FOR 6 FACTORS

Each error bar is constructed using the upper and lower quartiles.
PROPIDENTINS VS. X FOR 6 FACTORS
PROPIDENTINS VS. CAMOUFLAGE AT DIFFERENT HEIGHTS
HONEST ASSESSMENT APPROACH
USING TRAIN, VALIDATE (TUNE), AND TEST SUBSETS

Used in model selection and estimating its prediction error on new data

The Elements of Statistical Learning – Data Mining, Inference, and Prediction
Hastie, Tibshirani, and Friedman – 2001
(Chapter 7: Model Assessment and Selection)
R-SQUARE VS. NUMBER OF SPLITS
(FOR A RANDOM SPLIT INTO TRAIN, VALIDATE, & TEST)
Each split finds the cut point among all factors that creates the biggest difference in the means of the two partitions of the data.
DECISION TREE

Each split finds the cut point among all factors that creates the biggest difference in the means of the two partitions of the data.

Can be interpreted as a series of nested “If” statements.
HONEST ASSESSMENT WHEN DATA MINING

SUBSET DATA TO CREATE TRAIN, VALIDATE(TUNE), & TEST GROUPS
USE VALIDATE(TUNE) GROUP TO PREVENT OVERFITTING DATA MINING MODELS

First 5 splits raise Val R^2 from 0 to 0.908
20 more splits to raise Val R^2 from 0.908 to 0.915
COMPARE SEVERAL MODELS
Logistic Regression, Partition with 5-Splits, Neural Network, & LASSO Binomial
Four Models
1. Logistic Regression
2. Partition with 5-Splits
3. Neural Network
4. LASSO Binomial
ACTUAL VS. PREDICTED PLOTS FOR TEST DATA ONLY

LOGISTIC REGRESSION PARTITION WITH 5-SPLITS
NEURAL NETWORK
LASOO BINOMIAL
WHY IS A SEQUENTIAL APPROACH SO USEFUL?

We wanted to not just do sensitivity analysis of the factors, but provide an interactive surrogate model of the long-running simulation so that analysts could evaluate “what if?” scenarios.

The problem was that the Computational Fluid Dynamics models we were looking to run could take a week on a single CPU or 12 hours on 50 CPU cluster. With on the order of 10 factors we expected to need to run on the order of 100 simulations. This meant it could be weeks or months before we could start our analysis.

Nested Latin Hypercube Designs gave us a way to start analyzing data after about the first 20% of the simulations were run. We also wanted to be able to run just enough simulations to achieve a surrogate model accuracy of 90%.
PROJECTIONS OF TRIAL LOCATIONS IN 2 FACTORS FOR A 10-FACTOR, 128-TRIAL, NESTED LATIN HYPERCUBE DESIGN* (NLHD) WITH 4 BLOCKS

Running totals of blocks are also Latin Hypercube Designs

*Generated with Matlab Code Received from Prof. Peter Qian of U of Wi.
WHY RUN SIMULATIONS IN SEQUENTIAL BLOCKS?

The point of running this sequence of blocks is to be able to evaluate the surrogate model after each stage to see how accurately it is predicting observed values of 3 sets of checkpoint trials. If it proves to be sufficiently accurate, then subsequent blocks of simulation trials need not be run.

Without the NLHD approach one has to choose the “right” size space-filling design in order to get useful results. If you choose too small a design, one has to start over with a larger design.
COMPARE RESPONSE SURFACES FOR FIT OF 16 VS. FIT OF 128 TRIALS (LEFT) AND FOR FIT OF 64 VS. FIT OF 128 TRIALS (RIGHT)

Stage 1 fit of 16 trials colored green
Stage 4 fit of 128 trials colored brown
Stage 3 fit of 64 trials colored purple
ACCURACY OF SURROGATE PREDICTIONS FOR 3 GROUPS OF CHECKPOINTS YIELDING MARGINAL, MODERATE AND EXTREME EXTRAPOLATION

Trial Group vs. % Off Target as Sequential NLHD Blocks are Fit

- Chk.Pts. C - full range - 12
- Chk.Pts. B - 1/2 range - 12

% Off Target
- 4th Block - 64
- 3rd Block - 32
- 2nd Block - 16
- 1st Block - 16

% Off Target for Checkpoints NOT Included in Model Fit

% Off Target for Points Fit with Gaussian Process Modeling

% Off Target
- Block 1
- Blocks 1 & 2
- Blocks 1, 2 & 3
- Blocks 1, 2, 3 & 4

56
Checkpoint Groups A & B show diminishing return in prediction improvement for running past stage 3.
CONCLUSIONS SEQUENTIAL SPACE-FILLING DESIGNS

• NLHD designs can be run sequentially so that surrogate model accuracy can be evaluated after each block and decision made as to whether or not to move forward with the next block.

• Generally as more NLHD blocks are run, the surrogate model accuracy increases.

• Inclusion of extreme (full range) extrapolation checkpoints will expand interpolation volume of Kriging analysis – assuming Kriging analysis remains stable.

• Caveat: These conclusions were reached using a moderately complex transcendental function in lieu of a CFD simulation model that is believed to do a good job of stressing extrapolation with the surrogate model.
WHY USE DESIGN OF EXPERIMENTS METHODS WITH SIMULATION EXPERIMENTS?

Quicker answers, lower costs, solve bigger problems

• Obtain a fast surrogate model of the simulation
  • Individual simulations can run for hours, days, weeks
    • Computational Fluid Dynamics (CFD) or Simulation runs in real-time
  • Numbers of factors can be very large (100+)
  • Numbers of simulations needed can be large (thousands in many cases)
  • Simulations can be stochastic requiring many replications

• Surrogate model yields a fast approximation of the simulation
  • more rapidly answer “what if?” questions – *Instantaneous answer for any NEW scenario!*
  • do sensitivity analysis of the control factors
  • optimize multiple responses and make trade-offs

• By running sequences of designs one can be as *cost effective as possible*
  & *run no more trials than are needed* to get a useful answer

• By running efficient subsets of all possible combinations, one can – for the same resources and constraints – *solve bigger problems*