A General Pressure Generation Model for Granular Propellant Fires

Frederick Paquet, Hoi Dick Ng and Mario Paquet
General background

Solid propellants transform chemical energy into mechanical energy through the generation of high pressures caused by the completion of a combustion reaction in a limited volume.
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The high pressures are caused by:

- A solid to gas transformation in a limited volume
- An energy release (heat)
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Since applications vary widely in their requirements, propellants can exhibit a large array of combustion behaviors.
General background

- High explosives
- Explosive Gases & dusts
- Liquid fuels
- Solvents pools
- Dwellings

Burning velocity
General background

Heat release rate (area scaled) (MJ / s m²)

Mass burning rate (area scaled) (kg / s m²)

- Liquid fuels
- Polymers
- Woods
- Dust
- Gases
- Propellants
- High explosives
Previous work

An attempt was made by Graham to model explosion venting and define a critical vent area ratio by equating pressure rise and decrease terms:

\[
\frac{A_v}{S_B} = \frac{R T_B \rho \alpha}{M C_D A^* (A - B T_o)}
\]
Previous work

An attempt was made by Graham to model explosion venting and define a critical vent area ratio by equating pressure rise and decrease terms:

\[
\frac{A_v}{S_B} = \frac{RT_B \rho \alpha}{MC_D A^*(A - BT_o)}
\]

A number of empirical relations have been published for gas and dust equations (see NFPA 68 for examples).

- Gas explosions: \( A_v = C A_s P_{\text{red}}^{-1/2} \)

- Dust Explosions: \( A_v = 10^{-4} K_{st} V^{0.75} \left( \frac{P_{\text{max}}}{P_{\text{red}}} \right)^{1/2} \)

→ Deflagration index, \( K_{st} \), and reduced pressure, \( P_{\text{red}} \).
Theoretical background

Equation of state:

Nobel-Abel:

\[ P(V - b) = NRT \]

when

\[ X = cb \gg 0.01 \]

Most cases can use the ideal gas law:

\[ PV = NRT \]
Theoretical background

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Spatial pressure variation:

\[ \chi = \frac{m_{\text{air}} c_{\text{air}}}{\dot{m}_{\text{gen}} d} \]

Lumped parameters:

\[ \to \chi \gg 1 \]

<table>
<thead>
<tr>
<th>Case</th>
<th>( \chi ) (dimensionless)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7 L closed vessel with a fast burning propellant</td>
<td>0.9</td>
</tr>
<tr>
<td>0.7 L closed vessel with a slow burning propellant</td>
<td>5.6</td>
</tr>
<tr>
<td>60 L tank with a fast burning propellant</td>
<td>420</td>
</tr>
<tr>
<td>1800 L enclosure with a fast burning propellant</td>
<td>561</td>
</tr>
<tr>
<td>10^6 L enclosure with a fast burning propellant</td>
<td>675</td>
</tr>
</tbody>
</table>
Theoretical background

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\[ \chi = \frac{m_{air} c_{air}}{m_{gen} d} \]

Lumped parameters

\[ \rightarrow \chi \gg 1 \]

Compressibility:

Studied pressures below 10 kPa

\[ \rightarrow Ma < 0.30 \]

Bernoulli’s law:

\[ v = \left( \frac{2P}{\rho_{gas}} \right)^{1/2} \]

Other cases: isentropic eq.
Theoretical model

Mass balance: \( \dot{m}_e = \dot{m}_{\text{gen}} - \dot{m}_{\text{vent}} \)
Theoretical model

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Vented gases can be modelled by Bernoulli’s law and the ideal gas law:

\[
\dot{m}_{vent} = C_D A \left( \frac{2 \rho_{gas} m_e RT}{M_w V} \right)^{1/2}
\]
Theoretical model

Mass balance: $\dot{m}_e = \dot{m}_{gen} - \dot{m}_{vent}$

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$$\dot{m}_{vent} = C_D A \left( \frac{2 \rho_{gas} m_e R T}{M_w V} \right)^{1/2}$$

The end result is the following model:

$$\dot{m}_e + C_D A \left( \frac{2 \rho_{gas} R T}{M_w V} \right)^{1/2} m_e^{1/2} - \dot{m}_{gen} = 0 \quad \text{when} \ 0 \leq t \leq t_{burn}$$
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\]

\[
\dot{m}_e + C_D A \left( \frac{2\rho_{gas} R T}{M_w V} \right)^{1/2} m_e^{1/2} = 0 \quad \text{when} \ t > t_{burn}
\]
Testing

1800 L setup

Typical measurement
## Testing

<table>
<thead>
<tr>
<th>Propellant</th>
<th>Geometry</th>
<th>Heat of explosion (J/kg)</th>
<th>Composition</th>
</tr>
</thead>
<tbody>
<tr>
<td>SB1</td>
<td>Unitubular</td>
<td>3871</td>
<td>NC: 98% / Inert: 2%</td>
</tr>
<tr>
<td>SB2</td>
<td>Unitubular</td>
<td>3135</td>
<td>NC: 90% / Inert: 10%</td>
</tr>
<tr>
<td>DB1</td>
<td>Cord</td>
<td>5392</td>
<td>NC: 60% / NG: 39% / Inert: 1%</td>
</tr>
<tr>
<td>DB2</td>
<td>Unitubular</td>
<td>4490</td>
<td>NC: 73% / NG: 25% / Inert: 2%</td>
</tr>
</tbody>
</table>

![Diagram of propellant geometries](image-url)
Flame propagation – Induction time

![Graph showing flame propagation and induction time](image)

- 60L
- 1800L

$t_{ind} (s)$ vs. Test
Data analysis

Best statistical model: \( P_{\text{max}} = \frac{194.6 m^{0.97}}{A^{1.34}} \) (with \( r^2 = 0.91 \))
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Departure from theoretical form suggests: \( C_D \approx A^{1/2} \)
Starting with the previously derived mass balance:

\[ \dot{m}_e + C_D A \left( \frac{2 \rho_{\text{gas}} R T}{M_w V} \right)^{1/2} m_e^{1/2} - \dot{m}_{\text{gen}} = 0 \quad \text{when} \quad 0 \leq t \leq t_{\text{burn}} \]
Numerical model

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An energy balance is also considered:

\[ \Delta T = \frac{\dot{m}_{\text{gen}} E \Delta t - C_v \dot{m}_{\text{vent}} T \Delta t}{C_v (m_{\text{air}} + \rho_{\text{gas}} V)} \]
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Gas density obtained through EOS: \( \rho_{gas} = \frac{P M_w}{R T} \)
Starting with the previously derived mass balance:

\[ \dot{m}_e + C_D A \left( \frac{2 \rho_{gas} RT}{M_w V} \right)^{1/2} m_e^{1/2} - \dot{m}_{gen} = 0 \quad \text{when} \quad 0 \leq t \leq t_{burn} \]

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Gas density obtained through EOS: \[ \rho_{gas} = \frac{PM_w}{RT} \]

Here, the mass generation rate can be approximated by:

\[ \dot{m}_{gen} = \frac{m}{t_{max} - t_{ind}} \]
Numerical model

<table>
<thead>
<tr>
<th>Volume ($m^3$)</th>
<th>Min. Density ($kg/m^3$)</th>
<th>Max. Temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
<td>0.74</td>
<td>452</td>
</tr>
<tr>
<td>1.8</td>
<td>0.48</td>
<td>823</td>
</tr>
<tr>
<td>100</td>
<td>0.18</td>
<td>1798</td>
</tr>
</tbody>
</table>
The gas density is estimated as

\[ \rho_{\text{gas}} = 0.47 V^{-0.19} \]

Subst. in theoretical model

\[ P_{\text{max}} = \frac{1.06 m^2 V^{0.19}}{A^{2.66}} \]
Comparing the scale dependent model and measured results:

![Graph comparing predicted and measured max. pressure](image)

→ Loss in quality due to the uncertainty in estimating $m_{gen}$.
Future work

- Application to safety aspects
  - Quantities - distances relations, uniforms, building - process design, ...

- Better estimate of the mass combustion rate

- Application to “non dry” (or green) product

- Application to stick geometries (e.g. rocket motors)

- Application to cases at higher densities (higher pressures)
Acknowledgements

- General Dynamics OTS Canada - Valleyfield
  - Use of their burning ground and equipments
  - Donation of the propellant samples (value ≈ 125,000$)

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  - Work partly supported by Engage Grant EGP 415114-11