Higher-Order Finite-Element Analysis for Fuzes Subjected to High-Frequency Environments

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Outline

- Background
- Comparison of first-order and higher-order elements in explicit solid dynamics
  - Finite-deformation plasticity
  - Wave propagation
- Summary and Conclusions
Impact generates wave front

Reflections disrupt wave front

Background

Fuze components subjected to high-frequency waves

steel case

Al housing

circuit boards

electronics

potting

fuze

target
Background

- Lagrangian finite-element codes are industry standard for analysis of wave propagation
  - Explicit time integration by central differences
  - First-order elements
  - Computations often can’t resolve high-frequency modes, resulting in spurious oscillations (Gibbs’ phenomenon)
  - Artificial viscosity damps oscillations and high-frequency modes

- Objective is to improve the accuracy of Lagrangian computations of wave propagation
  - Systematic survey of numerical methods uncovered advantages of higher-order (> 2nd order) elements
  - Higher-Order elements formulated and added to the EPIC code
Background

- Higher-order elements used successfully for years in CFD
- Higher-order elements not used for solid mechanics because:
  - Computational efficiency of explicit schemes historically equated to minimizing the floating-point operations (FLOPS) in evaluation of internal-force term, and FLOPs increase with element order.
  - Greater complexity of curved-surface contact algorithms
  - Decades of research invested in various formulaic tradeoffs between locking and zero-energy modes of first-order elements
  - Mass lumping of 2nd-order serendipity elements yields vertex nodes with zero or negative:
    - Masses
    - Nodal forces due to uniform external traction
  - Lack of meshing and visualization software for higher orders
Finite-deformation plasticity

Square copper rod impacting a rigid surface at 200 m/s

- Symmetric order-1 tetrahedra
- Non-symmetric order-1 tetrahedra
- Order-1 hexahedra (Flanagan-Belytschko)
Finite-deformation plasticity

Square copper rod impacting a rigid surface at 200 m/s

order-2 hexahedra  order-3 hexahedra  order-4 hexahedra

plastic strain

2.1
1.7
1.3
0.9
0.5
0.1
Finite-deformation plasticity

No volumetric locking
Wave propagation in 2-D axisymmetry

Baseline mesh of simple part loaded by a pulse

- Monitored node near top
- Monitored node in base
- Element size: 1x1 cm

4340 steel:
- \( c_1 = 5845 \text{ m/s} \)
- \( c_2 = 4451 \text{ m/s} \)

- Pulse
- \( v_z(t) \)
- 5 m/s
- 2 \( \mu \text{s} \)
Wave propagation in 2-D axisymmetry
Wave propagation in 2-D axisymmetry

Velocities of node in base at equal mesh refinement
Wave propagation in 2-D axisymmetry

Velocities of node in base at equal mesh refinement
Wave propagation in 2-D axisymmetry

Velocities of node in base at equal mesh refinement

![Graph showing node velocity vs. time](image)

- **exact**
- **order = 20 (1x)**
- **order = 1 (20x)**

**Axes:**
- **Vertical:** node velocity (m/s)
- **Horizontal:** time (mcs)
Wave propagation in 2-D axisymmetry

Summary of errors at node in base (0 - 12 µs)
Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with element order

![Graph showing node velocity vs time with two curves representing different element orders.](image)
Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with element order
Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with element order

- Order = 20
- Order = 5
Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with element order

![Graph showing convergence of node velocity with different element orders](image-url)
Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with element order
Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with refinement of first-order quads
Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with refinement of first-order quads

![Graph showing node velocity over time](image-url)
Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with refinement of first-order quads
Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with refinement of first-order quads
Wave propagation in 2-D axisymmetry

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Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with refinement of first-order quads
Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with refinement of first-order quads
Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with refinement of first-order triangles

![Graph showing convergence of top-node velocity with refinement of first-order triangles](image-url)
Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with refinement of first-order triangles
Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with refinement of first-order triangles

![Graph showing node velocity vs. time with two curves for different triangle orders.](image-url)
Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with refinement of first-order triangles
Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with refinement of first-order triangles
Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with refinement of first-order triangles
Wave propagation in 2-D axisymmetry

Convergence of top-node velocity with refinement of first-order triangles
Wave propagation in 2-D axisymmetry

Comparison of velocity convergence with order and refinement
Wave propagation in 2-D axisymmetry

Comparison of velocity convergence with order and refinement

- order = 20 (1x)
- order = 5 (1x)
- order = 1 (5x)

Node velocity (m/s) vs. time (mcs)
Wave propagation in 2-D axisymmetry

Comparison of velocity convergence with order and refinement
Wave propagation in 2-D axisymmetry

Comparison of velocity convergence with order and refinement
Wave propagation in 2-D axisymmetry

Summary of errors in velocity at node near top

Errors relative to data from 20th-order elements for t = 0-100 μs

(normalized L1 error in velocity) vs. mean node spacing (cm)

(1)
Wave propagation in 2-D axisymmetry

Summary of errors in velocity at node near top

(1) Errors relative to data from 20th-order elements for t = 0-100 µs
(2) Intel Core i7: 2.93 GHz
15 GB RAM
Summary of errors in velocity at node near top

Wave propagation in 2-D axisymmetry

(1) Errors relative to data from 20th-order elements for t = 0-100 µs
Wave propagation in 3D

Baseline mesh of simple part loaded by a pulse

- Monitored node (same location as 2D)
- Mesh on plane of symmetry identical to 2-D mesh
- Model differs from 2D due to facets along hoop direction

$v_z(t)$

$v_z(t)$

5 m/s

2 μs
Wave propagation in 3D

Comparison of 2-D and 3-D node velocities

![Graph showing comparison of 2D and 3D node velocities](image)
Wave propagation in 3D

Convergence of top-node velocity with element order

![Graph showing convergence of top-node velocity with element order](image)
Wave propagation in 3D

Convergence of top-node velocity with element order
Wave propagation in 3D

Convergence of top-node velocity with element order

![Graph showing node velocity with time](image-url)
Wave propagation in 3D

Convergence of top-node velocity with element order

![Graph showing node velocity vs. time with order labeled as 7 and 6]
Wave propagation in 3D

Convergence of top-node velocity with refinement of first-order hexes
Wave propagation in 3D

Convergence of top-node velocity with refinement of first-order hexes
Wave propagation in 3D

Convergence of top-node velocity with refinement of first-order hexes

![Graph showing node velocity (m/s) over time (mcs)]
Wave propagation in 3D

Convergence of top-node velocity with refinement of first-order hexes
Wave propagation in 3D

Convergence of top-node velocity with refinement of first-order hexes

Diagram: Graph showing node velocity (m/s) over time (mcs) for two cases: order = 7 and first-order hexes (10x).
Wave propagation in 3D

Convergence of top-node velocity with refinement of first-order hexes

![Graph showing node velocity over time with convergence for order 7 and first-order hexes (12x)]
Wave propagation in 3D

Convergence of top-node velocity with refinement of first-order hexes

![Graph showing node velocity vs. time for different order solutions](image_url)
Wave propagation in 3D

Convergence of top-node velocity with refinement of first-order hexes
Wave propagation in 3D

Comparison of convergence with order and refinement

![Graph showing comparison of convergence with different order and refinement. The graph plots node velocity (m/s) against time (mcs). The lines represent different orders: order = 7, order = 3, and first-order hexes (3x).]
Wave propagation in 3D

Comparison of convergence with order and refinement

![Graph showing node velocity (m/s) vs. time (mcs)]

- order = 7
- order = 4
- first-order hexes (4x)
Wave propagation in 3D

Comparison of convergence with order and refinement
Wave propagation in 3D

Summary of velocity errors at monitored node

- First-order hexahedra
- First-order quads
- 3-D higher-order elements
- 2-D higher-order elements

(1) Errors relative to data from 7th-order elements for t = 0-100 µs
Wave propagation in 3D

Summary of velocity errors at monitored node

(1) Errors relative to data from 7th-order elements for t = 0-100 µs

(2) AMD Opteron:
- 2.31 GHz
- 15.7 GB RAM
Wave propagation in 3D

Summary of velocity errors at monitored node

(1) Errors relative to data from 7th-order elements for t = 0-100 µs
Summary and conclusions

- Analysis of wave propagation is essential to fuze design
- 1D, 2D and 3D higher-order elements have been formulated and implemented in EPIC
- The higher-order elements show no signs of volumetric locking
- Accuracy of higher-order elements is compared to standard first-order elements in simulations of wave propagation. Higher-order elements provide much greater accuracy at equal:
  - Mesh refinement
  - Computing time
  - Allocated memory
Acknowledgment

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