The Three-Phase Optimal Design Test Meets Reality: Lessons Available, Part Two

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Outline

- Our Example Problem
- The Three-Phase Optimal Design Test
- Issues and How to Cope With Them
  - Data Range
  - Limited Precision
  - Specified vs. Actual Test Points
  - End of Test
Our Example Problem

- Fictitious Weapon: Electro-Magnetic Pulse Against Thoroughly Hostile Yetis
  - Two high-voltage electrodes
  - Separated by stack of insulating blankets
  - Thicker stack $\rightarrow$ better chance of enough insulation between electrodes $\rightarrow$ better chance that charge does not bleed off slowly $\rightarrow$ better chance of electrical discharge when needed
  - Need thickness of stack required to give 99.99% chance of discharge at 95% confidence level
Our Example Problem (2)

Electrode

Blankets

 Thickness

Electrode

Target

EMPATHY Aperture

(Picture of airplane with opening on top)

(Picture of yeti)
The Three-Phase Optimal Design Test

- We have an input
  - Varies continuously – thickness of stack

- We have an output
  - One or zero – success or failure – on or off – discharge or no discharge
  - Probabilistic function of input
    - The same input can give different outputs in different tests
    - Probability of a one increases as input increases
The Three-Phase Optimal Design Test (2)

- Invented by
  - Jeff Wu of Georgia Institute of Technology
  - Yubin Tian of Beijing Institute of Technology

- Published in the Journal of Statistical Planning and Inference, 2013
  - http://dx.doi.org/10.1016/j.jspi.2013.10.007
The Three-Phase Optimal Design Test (3)

- Phase I: Find the mean
  - Step I1: Obtain success and failure results
  - Step I2: Get an overlapping result
  - Step I3: Enhance the overlapping result

- Phase II: Optimize the mean and standard deviation

- Phase III (optional): Test at desired probability level to reduce uncertainty
The Three-Phase Optimal Design Test (4)

- Assumes probability curve follows normal distribution

- Requires starting values:
  - Approximately lower and upper bounds of range
  - Approximately standard deviation of probability curve
Our Example Problem (3)

- Simulations show:
  - 1.6-meter stack of blankets is not enough insulation — no discharge
    - Lower end of “initial guess” interval
  - 1.8-meter stack of blankets is enough insulation — discharge
    - Upper end of “initial guess” interval

- Estimated Standard Deviation
  - Should be less than one sixth of range
  - We use 0.015 meters
Our Example Problem (4)

- Discharge
- No Discharge

**EMPATHY System Test**

\[
\{\mu_{lo}, \mu_{hi}, \sigma_g | n_1, n_2, n_3 | p, \text{res} \}
\]

\[
\{1.6, 1.8, 0.015 | 9, 27, 0 | 0, 0 \}
\]
Our Example Problem (5)

Nominal Values:
Mu = 1.750
Sigma = 0.050

Final Calculated Values:
Mu = 1.757
Sigma = 0.029
Our Example Problem (6)
Issues

- Data Range
- Limited Precision
- Specified vs. Actual Test Point Values
- End of Test
Data Range

❖ Issue:
  • Method is mathematical
    ◆ No knowledge of physical limitations on system
    ◆ Can specify unreasonable test points
      o Negative thickness of stack of blankets
      o Stack thickness beyond system capability

❖ Resolution:
  • Use common sense
Example (5)

- EMPATHY system testing:
  - If first several tests give “discharge” result:
    - Thickness of blanket stack decreases
    - Next test point requires negative thickness
      - Not physically real
  - If first several tests give “no discharge” result:
    - Diameter of EMPATHY case is 2.14 meters
    - Hard upper limit on blanket stack thickness
      - May result in system not meeting requirement
**Limited Precision**

- **Issue:**
  - 3POD method can specify test points to unlimited precision
  - Test articles cannot be built to unlimited precision

- **Resolution:**
  - Points close to optimal point are still good
  - Do your best
Three different things:

- Test point specification – result of 3POD method
- Test item fabrication – built to specified point at limited precision
- Test item measurement – may be more precise than test item fabrication ability
Limited Precision (3)

● Resolution (2)
  • Specify test points to 3POD recommended precision—do not round to specifiable precision
    ● Scatter will center around recommended point
  • Use measured values in 3POD method calculations
    ● Not specified values
    ● Not rounded measured values
    ● Not 3POD method’s recommended values
Example (6)

- EMPATHY blankets settle irregularly
  - Final thickness controlled only to $\pm 0.01$ m
    - One centimeter scatter on either side
  - Can be measured to $\pm 0.0005$ m
    - One-millimeter uncertainty overall
Example with Limited Precision

Limited Precision Test Points

EMPATHY System Test
\{\mu_{lo}, \mu_{hi}, \sigma_g | n_1, n_2, n_3 | p, res\}
\{1.6, 1.8, 0.015 | 10, 26, 0 | 0, 0\}
Example with Limited Precision

Overlaid with Nominal Test Points
Effect of Limited Precision

<table>
<thead>
<tr>
<th>Test</th>
<th>Nominal Value</th>
<th>Outcome</th>
<th>Limited Precision Value</th>
<th>Actual Value</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.65</td>
<td>No Discharge</td>
<td>1.65</td>
<td>1.652</td>
<td>No Discharge</td>
</tr>
<tr>
<td>2</td>
<td>1.75</td>
<td>No Discharge</td>
<td>1.75</td>
<td>1.746</td>
<td>No Discharge</td>
</tr>
<tr>
<td>3</td>
<td>1.8225</td>
<td>Discharge</td>
<td>1.82250</td>
<td>1.827</td>
<td>Discharge</td>
</tr>
<tr>
<td>4</td>
<td>1.78625</td>
<td>Discharge</td>
<td>1.78650</td>
<td>1.795</td>
<td>Discharge</td>
</tr>
<tr>
<td>5</td>
<td>1.76812</td>
<td>No Discharge</td>
<td>1.77050</td>
<td>1.777</td>
<td>Discharge</td>
</tr>
<tr>
<td>6</td>
<td>1.79075</td>
<td>Discharge</td>
<td>1.76150</td>
<td>1.770</td>
<td>Discharge</td>
</tr>
<tr>
<td>7</td>
<td>1.76362</td>
<td>Discharge</td>
<td>1.75800</td>
<td>1.750</td>
<td>No Discharge</td>
</tr>
<tr>
<td>Final Values</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mu</td>
<td>1.757</td>
<td>Sigma 0.029</td>
<td>Mu 1.754</td>
<td>Sigma 0.034</td>
<td></td>
</tr>
</tbody>
</table>
Limited Precision (4)

- Issue:
  - Standard deviation of distribution may be near limit of precision of creating test items
  - Specified test points in Phase I2 may all round to the same value, preventing overlap
Resolution:

• Alternative 1: Use “engineering judgment” to modify test points for Phase I2
  - If tests never achieve overlap, standard deviation is less than measurement precision

• Alternative 2: Add fictitious “test points” at changeover point to create artificial overlap
Example: Alternative 1

- Can build, measure only to 0.01:

<table>
<thead>
<tr>
<th>Test</th>
<th>Specified Test Point</th>
<th>Rounded Test Point</th>
<th>Actual Test Point</th>
<th>Test Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.65</td>
<td>1.65</td>
<td>1.65</td>
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<td>2</td>
<td>1.75</td>
<td>1.75</td>
<td>1.75</td>
<td>Discharge</td>
</tr>
<tr>
<td>3</td>
<td>1.7</td>
<td>1.70</td>
<td>1.70</td>
<td>No Discharge</td>
</tr>
<tr>
<td>4</td>
<td>1.725</td>
<td>1.72</td>
<td>1.72</td>
<td>No Discharge</td>
</tr>
<tr>
<td>5</td>
<td>1.735</td>
<td>1.74</td>
<td>1.74</td>
<td>No Discharge</td>
</tr>
<tr>
<td>6</td>
<td>1.7545</td>
<td>1.75</td>
<td>1.75</td>
<td>Discharge</td>
</tr>
<tr>
<td>7</td>
<td>1.7355</td>
<td>1.74</td>
<td>1.74</td>
<td>No Discharge</td>
</tr>
<tr>
<td>8</td>
<td>1.753</td>
<td>1.75</td>
<td>1.76</td>
<td>Discharge</td>
</tr>
<tr>
<td>9</td>
<td>1.737</td>
<td>1.74</td>
<td>1.73</td>
<td>...</td>
</tr>
</tbody>
</table>

Settling around two points
Example: Alternative 2

- Can build, measure only to 0.01:

<table>
<thead>
<tr>
<th>Test</th>
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<th>Rounded Test Point</th>
<th>Test Result</th>
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<td>1.737</td>
<td>1.74</td>
<td>Discharge</td>
</tr>
</tbody>
</table>
Specified vs. Actual Test Points

- **Issue:**
  - 3POD method
    - Assumes actual test points same as specified test points
    - Declares overlap based on test result without checking actual test point
  - 3POD algorithm may specify leaving Phase I2 without achieving overlap

- **Resolution:** Use common sense
Issue:

- 3POD method does not specify number of tests or ending criterion
- Number of tests often governed by economics and other factors
- “99.99% value” vs. “acceptable 99.99% value”
Resolution

- Specify ending criterion in advance: “99.99 percent probability, with 95 percent confidence, that a thickness of 2.14 meters will not allow a spark between electrodes”
- Continue testing until
  - Criterion is met
  - Criterion will still be met if next three* tests give less probable result

*arbitrary number
Issue:

- Calculation of 99.99%-at-95% point
  - Issue: Different methods give different results
    - Logit link vs. Probit link vs. other links
      - (define exact shape of probability curve)
    - Which do you believe?
If criterion is never met:

• More testing will tighten 95% confidence bounds

• Possibility that criterion cannot be met
  ♦ It may take 2.15 meters of blankets to prevent spark between the electrodes
End of Test (5)

• “Point of No Return”
  • Situation: Phase II
    o Predicted 99.99%-at-95% “threshold” point exceeds maximum value
    o Hard limit on number of tests possible
  • Suggestion: Predict test into the future
    o Assume no further anomalies
    o Determine whether remainder of test shots can bring threshold point down to an acceptable level
    o If not, consider declaring failure early and saving test resources
Conclusions

- 3POD method can be successfully applied to a “real-world” situation

- “Lessons Learned?”
  - Lessons are available
  - Learning them is everybody’s job
Any questions?