Gurney Velocity Relationships

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Outline

- Summary
- Gurney Theory
- Gurney Velocity Theories
- Solution to Gurney Velocity Theories
- Fragment Energy to Charge Energy Ratios
- Conclusions

Summary

Gurney Velocity is a measure of metal casing fragment velocity from a High Explosive detonation - Measurement of Gurney Velocity is difficult.
But Detonation Velocity and explosive density are easily measured

Can you link Gurney Velocity & Energy to the energy available from the charge, in this case the Hydrodynamic Work Energy?
This paper derives theoretical relationships from the Gurney theories of:
Kamlet & Finger, Cooper and Koch

Linking:

1. Gurney Velocity to Detonation Pressure and explosive density
2. Gurney Velocity to Detonation Velocity and explosive density
3. Gurney Energy to Hydrodynamic Work Energy
   Their ratio is just a function of explosive density

This paper theoretically derives 62% of the Hydrodynamic Work Energy as the Gurney Energy for typical High Explosives

Gurney Theory

Gurney theory (1943) links fragment velocity to explosive composition
Different equations for: cylinders, spheres, plates etc.
Gurney Velocity (VG) is only a function of composition & derived from trials

This paper links Gurney Velocity and Energy to:

- Detonation Velocity
- Hydrodynamic Work Energy
- Unreacted explosive density

Three Gurney Velocity theories were studied:

- Kamlet & Finger (79), Cooper (96) & Koch (2002)
- Solutions found are both simple and easy to use

Gurney Velocity Theories

1. Kamlet & Finger

In mixed units we have:

$$VG_X = 0.887.\ \varphi^{0.5}.\ \rho_X^{0.4}$$

Where:

$$\varphi = N\ M^{0.5}\ Q^{0.5}$$

Gives in SI:

$$VG_X = \varphi^{0.5}.\ \rho_X^{0.4}.\ (0.887\ .\ 10^{1.8})$$

2. Cooper $VG_X = D_X / 2.97$

3. Koch $VG_X = D_X / 3.08$

Definitions:

- D_X - Detonation Velocity of Explosive x (m/s)
- M - Molecular weight of gases (g/mol)
- N - Moles of gas products per gram of explosive (mol/g)
- φ - Kamlet chemical parameter ($m^{4.4}\ s^{-2}\ kg^{-0.8}$)
- Q - Heat of Detonation (cal/g)
- ρ_X - Density of unreacted explosive x (kg/m^3)
- VG_X - Gurney Velocity for explosive x (m/s)

Nomenclature

α_X - Ratio of Gurney Energy to Hydrodynamic Work Energy for explosive x
$\beta = 3.6732$ - Energy constant $(m^3/kg)^{0.24}$
D_X - Detonation Velocity of Explosive x (m/s)
EG_X - Gurney Energy for explosive x (J/kg)
EHW_X - Hydrodynamic Work Energy for explosive x (J/kg)
f_X - Velocity reduction coefficient for explosive x
$k = 15.58$ - Kamlet & Jacobs constant
M - Molecular weight of gases (g/mol)
N - Moles of gas products per gram of explosive (mol/g)
φ - Kamlet chemical parameter $(m^{4.4}\, s^{-2}\, kg^{-0.8})$
$P_{CJ\,X}$ - Chapman-Jouguet (CJ) Detonation Pressure for explosive x (Pa)
Q - Heat of Detonation (cal/g)
ρ_X - Density of unreacted explosive x (kg/m^3)
VG_X - Gurney Velocity for explosive x (m/s)
$z = 1.3183 = 1000^{(1-0.96)}$ - Unit conversion to SI (-)

Gurney Model Assumptions

- Detonation Energy is completely changed to the Kinetic Energy of the gases and Kinetic Energy of the metal fragments
- Energy used for case breakup is ignored
- During detonation, the explosive is transformed into stable homogeneous, gaseous products at high pressure
- Gaseous products expand with uniform density and a linear velocity gradient
- Detonation Energy of the explosive is changed into Kinetic Energy, until the fragments reach a steady state velocity, from which Gurney velocity is calculated

Gurney Velocities - Symmetric

Key

v = Metal Fragment Velocity

VG = Gurney Velocity

E = Gurney Energy

So: VG = √(2E)

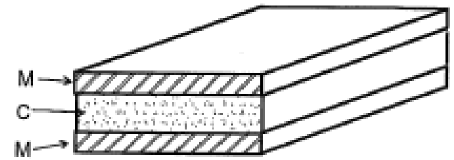
M = Metal Mass

C = Charge Mass

N = Tamper Mass

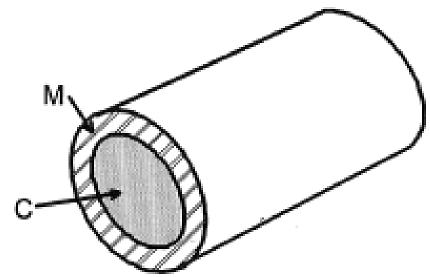
a) Symmetric Sandwich

$$\frac{v}{VG} = \left(2\frac{M}{C} + \frac{1}{3}\right)^{-1/2}$$



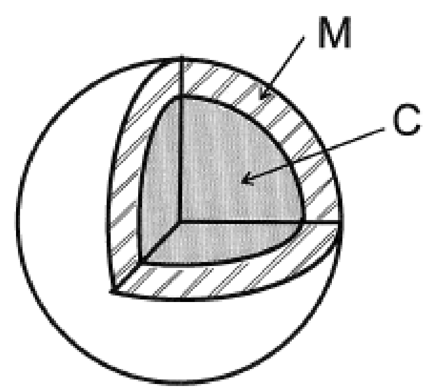
b) Cylinder

$$\frac{v}{VG} = \left(\frac{M}{C} + \frac{1}{2}\right)^{-1/2}$$



c) Sphere

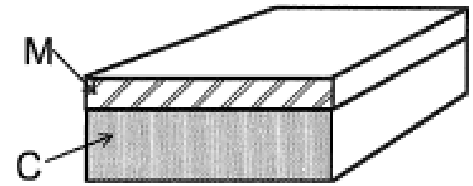
$$\frac{v}{VG} = \left(\frac{M}{C} + \frac{3}{5}\right)^{-1/2}$$



Gurney Velocities - Asymmetric

a) Open Face Sandwich

$$\frac{V}{VG} = \left[\frac{1 + \left(1 + 2\frac{M}{C}\right)^3}{6\left(1 + \frac{M}{C}\right)} + \frac{M}{C} \right]^{-1/2}$$

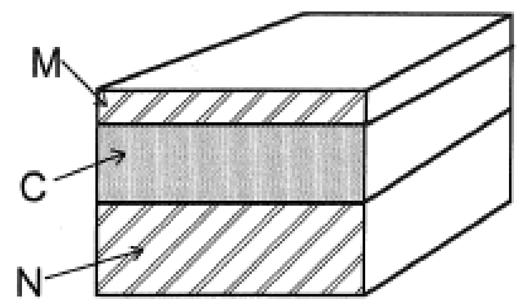


b) Asymmetric Sandwich

$$\frac{V}{VG} = \left[\frac{1 + A^3}{3(1 + A)} + \frac{N}{C}A^2 + \frac{M}{C} \right]^{-1/2}$$

where:

$$A = \frac{1 + 2\ M/C}{1 + 2\ N/C}$$



Hydrodynamic Work Energy

From the energy available to do work, we derive an isentropic expansion giving the energy available from the P_{CJ} to ambient pressure, this is the Hydrodynamic Work Energy.

From Cooper[3]:

$$EHW_X = \int_{P\,CJ}^{P\,AMB} P\,(V)_S \;.\; dV \qquad (1)$$

And:

$$EHW_X = 0.5 \;.\; P_{CJ\,X} \,/\, \rho_{CJ} \qquad (2)$$

But also from Cooper we have a good approximation of ρ_{CJ} as:
(Here, z is a constant to convert units from cgs to SI).

$$\rho_{CJ} = 1.386z \;.\; \rho_X^{\;0.96} \qquad (3)$$

So:

$$EHW_X = 0.27366 \;.\; P_{CJ\,X} \,/\, \rho_X^{\;0.96} \qquad (4)$$

Only explosive density (ρ_X) and $P_{CJ\,X}$ are required to give a good estimate of the available work energy.

Kamlet & Finger's Gurney Velocity with CJ-Detonation Pressure

From Kamlet & Finger [2] and adjusted to SI units we have:

$$VG_X = \varphi^{0.5} . \rho_X^{0.4} . (0.887 . 10^{1.8}) \quad (5)$$

And from Kamlet & Jacobs [6] in SI units we have:

$$\varphi = P_{CJ\,X} / (100k . \rho_X^{2}) \quad (6)$$

Substituting (6) into (5) gives:

$$VG_X = P_{CJ\,X}^{0.5} / (10.k^{0.5} . \rho_X) . \rho_X^{0.4} . (0.887 . 10^{1.8}) \quad (7)$$

$$= (P_{CJ\,X}^{0.5} / \rho_X^{0.6}) . (0.887 . 10^{1.8}) / (10 . k^{0.5}) \quad (8)$$

Now k = 15.58 [6], a constant so:

$$VG_X = (P_{CJ\,X}^{0.5} / \rho_X^{0.6}) . (0.887 . 10^{1.8}) / (10 . 15.58^{0.5}) \quad (9)$$

$$\boxed{VG_X = 1.4179 . P_{CJ\,X}^{0.5} / \rho_X^{0.6}} \quad (10)$$

From Dobratz [8]: $P_{CJ\,TNT}$ = 21GPa and ρ_{TNT} = 1,640kg/m^3

Giving the Gurney Velocity for TNT:

$$VG_{TNT} = 1.4179 . 21x10^{9\ 0.5} / 1640^{0.6} = 2,420 \text{ m/s} \quad (11)$$

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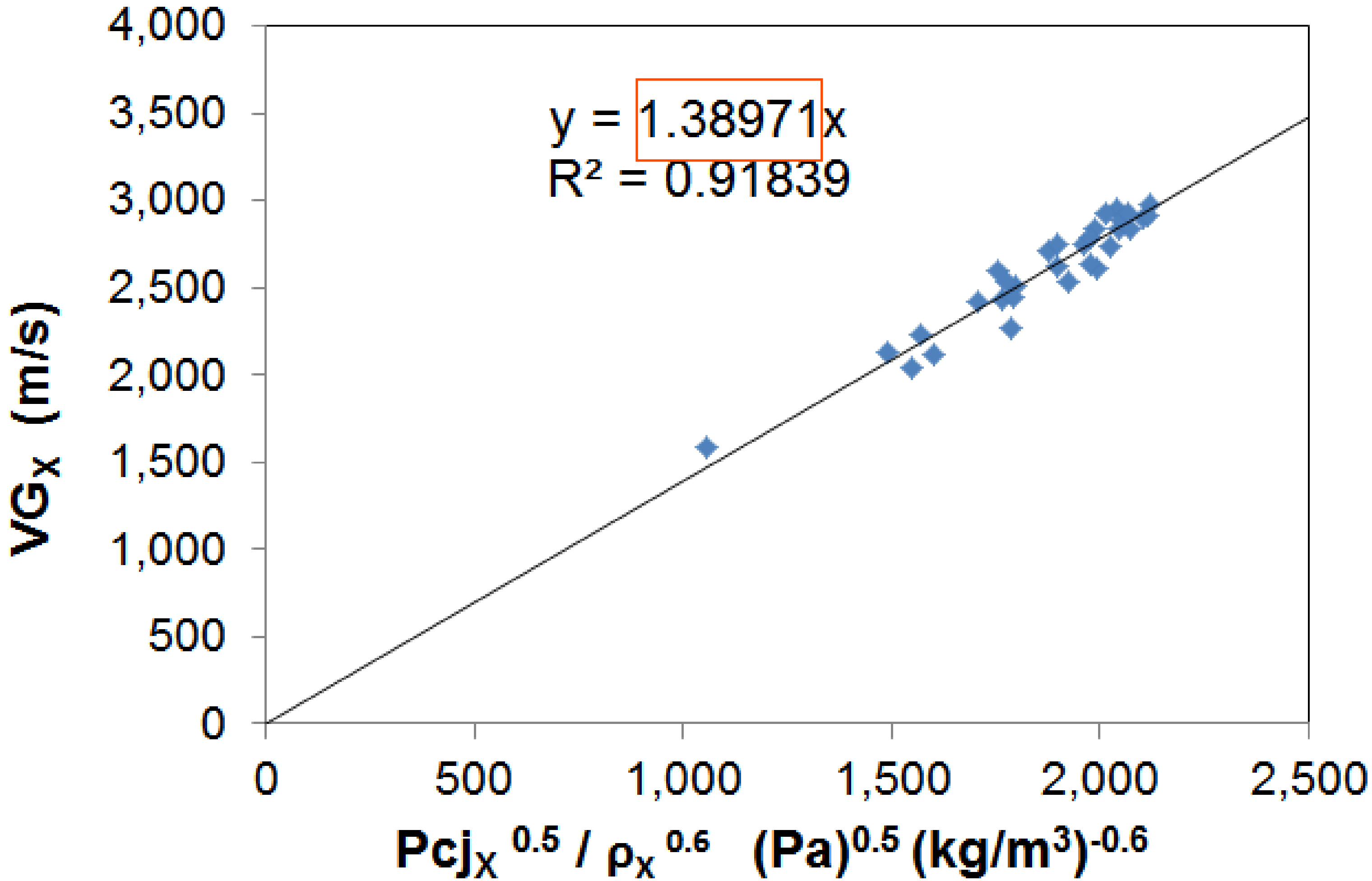

Figure 1. Experimental Gurney Data to Equation 10.

$$VG_X = 1.4179 \, . \, P_{CJ\ X}{}^{0.5} / \rho_X{}^{0.6} \qquad (10)$$

Gradient Error = 1.38971/1.4179 = 0.98

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Kamlet & Finger's Gurney Velocity from Detonation Velocity

Restating (5) from Kamlet & Finger [2] and adjusted to SI units we have:

$$VG_X = \varphi^{0.5} . \rho_X^{0.4} . (0.887 . 10^{1.8}) \qquad (5)$$

From Kamlet & Jacobs [6] after converting to SI, we have:

$$D_X = A. \varphi^{0.5} . (1+B. \rho_X /1000).10^3 \qquad (12)$$

$$\varphi^{0.5} = D_X / (A. (1+B. \rho_X /1000). 10^3) \qquad (13)$$

Thus:

$$VG_X = (D_X . \rho_X^{0.4}) / (1+B. \rho_X /1000) . ((0.887 . 10^{1.8}) / (A. 10^3)) \qquad (14)$$

Now A = 1.01 a constant [6] & B = 1.30 a constant [6], so that:

$$\boxed{VG_X = 0.0554 \ (D_X . \rho_X^{0.4}) / (1 + 1.3 \rho_X /1000)} \qquad (15)$$

From Dobratz [8] : D_{TNT} = 6,940m/s and ρ_{TNT} = 1,640kg/m^3

Giving the Gurney Velocity for TNT:

$$VG_{TNT} = 2{,}372 \text{m/s} \qquad (17)$$

This Gurney Velocity lies within 2% of the solution value from (11)

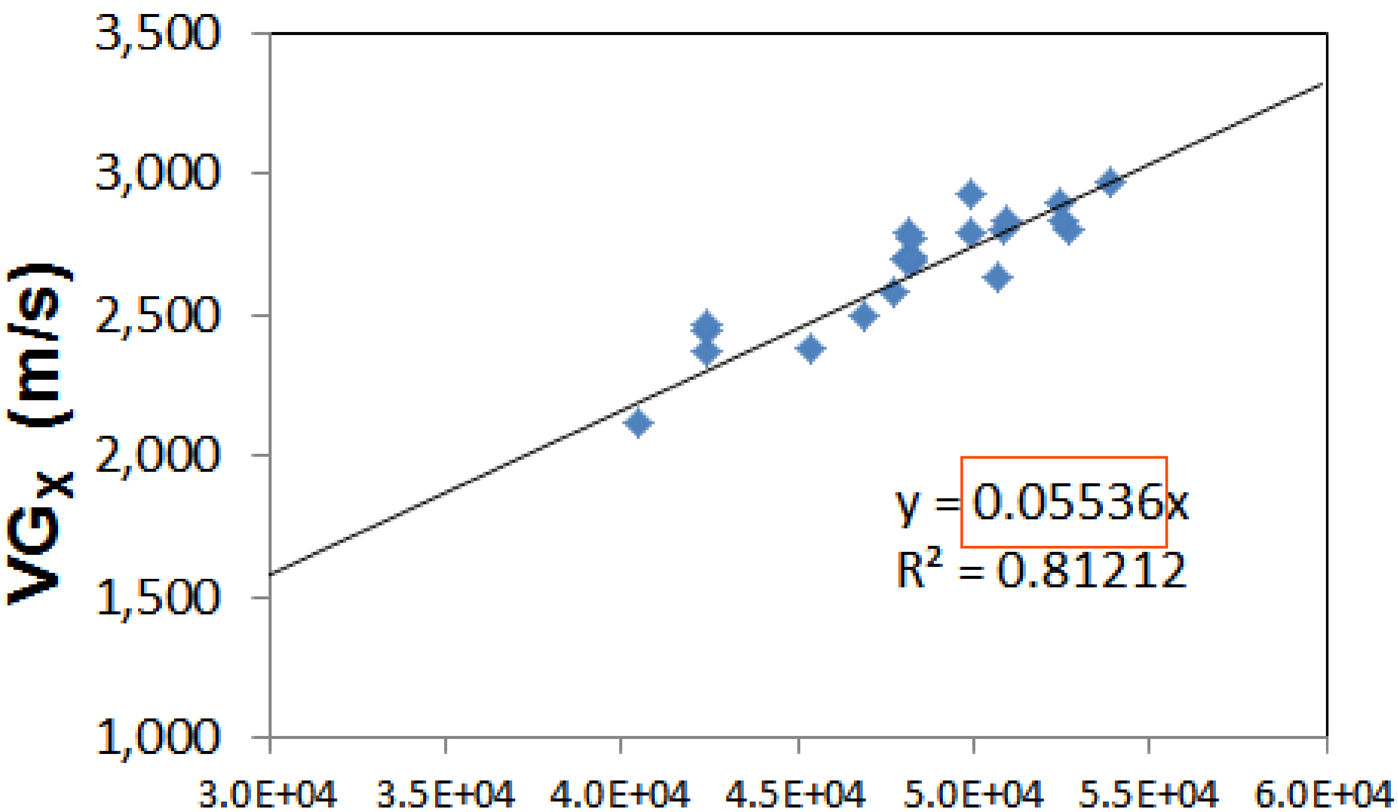

Figure 2. Experimental Gurney Data to Equation 15.

$$VG_X = 0.0554 \, (D_X \, . \, \rho_X^{\,0.4}) \, / \, (1 + 1.3 \, \rho_X /1000) \qquad (15)$$

Gradient Error = 0.05536/0.0554 = 0.999

Kamlet & Finger's CJ-Pressure with Detonation Velocity

Equating (10) & (15) gives:

$$(P_{CJ\,X}{}^{0.5} / \rho_X{}^{0.6}) \;.\; 1.4179 = D_X \;.\; \rho_X{}^{0.4} \;/\; (1 + 1.3\rho_X/1000) \;.\; 0.05541 \qquad (18)$$

$$P_{CJ\,X}{}^{0.5} = (D_X \;.\; \rho_X) \;/\; (1 + 1.3\,\rho_X/1000) \;.\; (0.05541 / 1.4179) \qquad (19)$$

$$P_{CJ\,X}{}^{0.5} = 0.03908.\, D_X \;.\; \rho_X \;/\; (1 + 1.3\,\rho_X/1000) \qquad (20)$$

Using values from Dobratz [8] for TNT :

$$P_{CJ\,TNT} = 20.2\text{GPa} \qquad (21)$$

As expected this is close to the value from Dobratz [8] of 21GPa, within 4%.

Gurney Velocity from Cooper and Koch Theories

From Cooper [3], we have simply:

$$VG_X = D_X / 2.97 \qquad (22)$$

And more recently from Koch [4] we have:

$$VG_X = (3\sqrt{3})\, D_X / 16 = D_X / 3.08 \qquad (23)$$

Generalising (22) & (23) gives:

$$\boxed{VG_X = D_X / f_X} \qquad (24)$$

Where coefficient $f_X = 2.97$ estimated by Cooper from trials and $f_X \approx 3.08$, estimated by Koch from theory. We now define an f_X which itself is also a function of the explosive material. Restating (15) as:

$$VG_X = 0.0554\ (\, D_X\ .\ \rho_X^{\ 0.4}\,)\ /\ (1 + 1.3\ \rho_X\ /1000) \qquad (15)$$

Substituting for (24) gives:

$$\boxed{f_X = 18.0467\ (1 + 1.3\ \rho_X\ /1000)\ /\ \rho_X^{\ 0.4}} \qquad (27)$$

From Dobratz [8] : $\rho_{TNT} = 1{,}640 kg/m^3$. So for TNT :

$$\boxed{f_{TNT} = 2.926} \qquad (29)$$

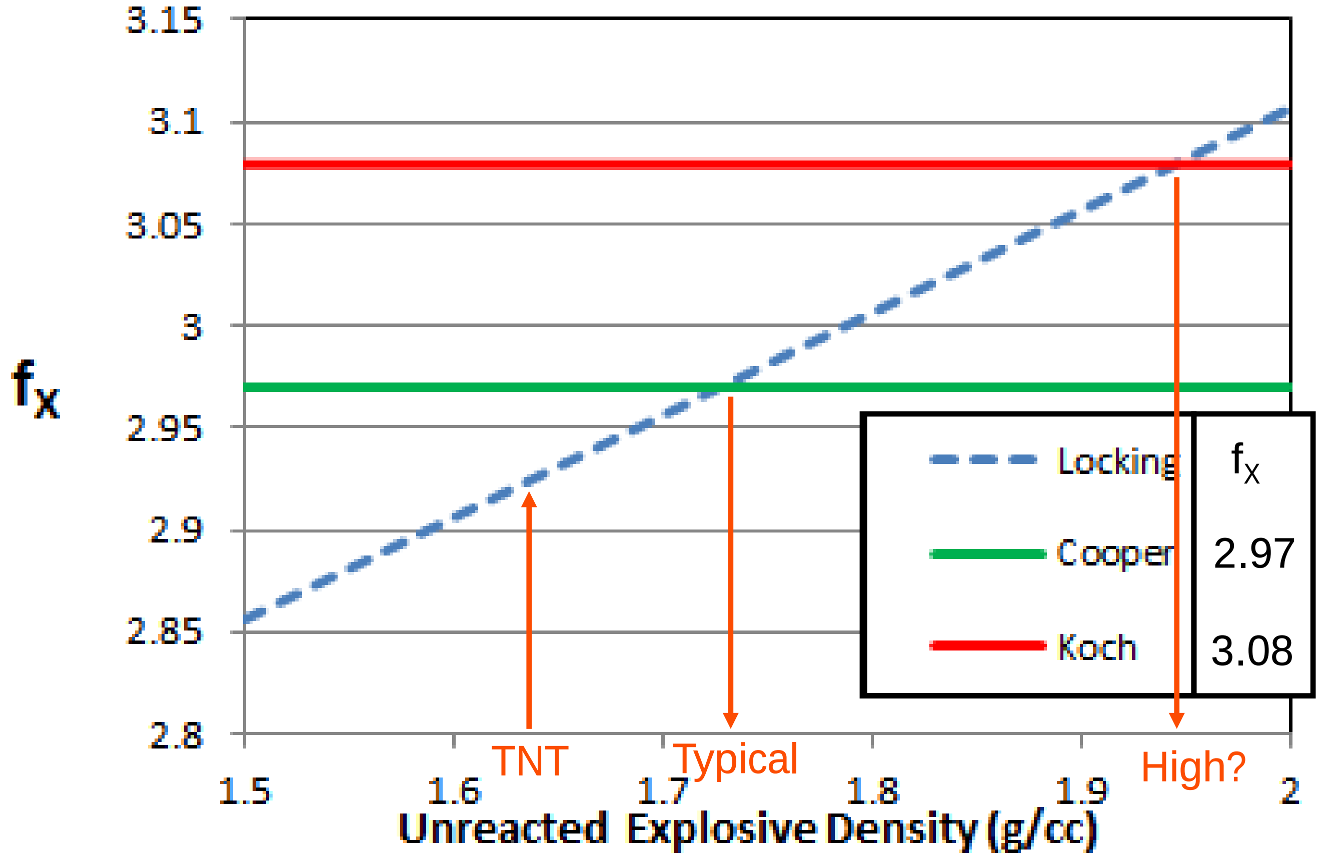

Figure 3. Plot of f_X with Explosive Density for Gurney Theories

$$f_X = 18.0467\ (1 + 1.3\ \rho_X / 1000)\ /\ \rho_X^{\ 0.4} \qquad (27)$$

$$VG_X = D_X\ /\ f_X \qquad (24)$$

Cooper's Data Table

TABLE I. EXPERIMENTAL AND CALCULATED GURNEY DATA

Explosive	Density (g/cc)	Det Velocity Trial (km/s)	Gurney Velocity Trial (km/s)	Cooper Gurney Calc (km/s)	Cooper Gurney Error (%)	Locking f_X Calc (-)	Locking Gurney Calc (km/s)	Locking Gurney Error (%)
Comp A3	1.590	8.14	2.630	2.741	4.2	2.901	2.806	6.7
Comp B	1.710	7.89	2.700	2.657	1.6	2.961	2.665	1.3
Comp B	1.717	7.91	2.790	2.663	4.5	2.965	2.668	4.4
Comp B	1.717	7.91	2.710	2.663	1.7	2.965	2.668	1.5
Comp B	1.720	7.92	2.680	2.667	0.5	2.966	2.670	0.4
Comp B	1.720	7.92	2.700	2.667	1.2	2.966	2.670	1.1
Comp B	1.720	7.92	2.710	2.667	1.6	2.966	2.670	1.5
Comp B	1.720	7.92	2.770	2.667	3.7	2.966	2.670	3.6
Comp C3	1.600	7.63	2.680	2.569	4.1	2.906	2.626	2.0
Cyclotol 75/25	1.754	8.25	2.790	2.778	0.4	2.983	2.765	0.9
H-6	1.760	7.90	2.580	2.660	3.1	2.986	2.645	2.5
HMX	1.835	8.83	2.800	2.973	6.2	3.024	2.920	4.3
LX-14	1.890	9.11	2.970	3.067	3.3	3.051	2.985	0.5
Octol 75/25	1.810	8.48	2.800	2.855	2.0	3.011	2.816	0.6
Octol	1.821	8.51	2.830	2.865	1.2	3.017	2.821	0.3
PBX 9404	1.840	8.80	2.900	2.963	2.2	3.026	2.908	0.3
PBX 9502	1.885	7.67	2.377	2.582	8.6	3.049	2.516	5.8
PETN	1.760	8.26	2.930	2.781	5.1	2.986	2.766	5.6
RDX	1.770	8.70	2.830	2.929	3.5	2.991	2.908	2.8
Tacot	1.610	6.53	2.120	2.199	3.7	2.911	2.243	5.8
Tetryl	1.620	7.57	2.500	2.549	2.0	2.916	2.596	3.8
TNT	1.630	6.86	2.370	2.310	2.5	2.921	2.348	0.9
TNT	1.630	6.86	2.440	2.310	5.3	2.921	2.348	3.8
TNT	1.630	6.86	2.460	2.310	6.1	2.921	2.348	4.5
Tritonal 80/20	1.720	6.70	2.320	2.256	2.8	2.966	2.259	2.6
Mean	1.727				3.3	2.970		2.7

Energy Ratios from Cooper and Koch Theories #1

From Gurney theory [1] we have:

$$EG_X = 0.5.\ VG_X{}^2 \qquad (30)$$

Defining α_X so that:

$$EG_X = \alpha_X .\ EHW_X \qquad (31)$$

From Kamlet & Jacobs [6] in SI units we have:

$$\varphi = P_{CJ\ X} / (100k .\ \rho_X{}^2) \qquad (32)$$

From (24) & (30) we then get:

$$EG_X = 0.5.\ D_X{}^2 / f_X{}^2 \qquad (33)$$

And using (2):

$$EG_X / EHW_X = \alpha_X = (0.5.\ D_X{}^2 / f_X{}^2) / (0.5.\ P_{CJ\ X} / \rho_{CJ}) \qquad (34)$$

So:

$$\alpha_X = D_X{}^2 .\ \rho_{CJ} / (\ f_X{}^2 .\ P_{CJ\ X}\ ) \qquad (35)$$

Energy Ratios from Cooper and Koch Theories #2

From Kamlet & Jacobs [6] after converting to SI, we have:

$$D_X = A.\ \varphi^{0.5}\ .\ (1+B.\ \rho_X /1000).10^{3} \qquad (36)$$

From (35):

$$\alpha_X = A^2.\ \varphi\ (1+B.\ \rho_X /1000)^2.10^{6}.\ \rho_{CJ} / (f_X^{2}\ .\ P_{CJ\ X}) \qquad (38)$$

From (32):

$$P_{CJ\ X} = \varphi\ .\ k\ .\ \rho_X^{2}.100 \qquad (39)$$

So:

$$\alpha_X = A^2 (1+B.\ \rho_X /1000)^2.10^{4}.\ \rho_{CJ} / (f_X^{2}.\ k.\ \rho_X^{2}) \qquad (41)$$

$$\alpha_X = 1.01^2 (1+1.3.\ \rho_X /1000)^2 .10^{4}\ .\ 1.386.\ 1.3183.\ \rho_X^{-1.04} / (\ f_X^{2}\ .\ 15.58) \qquad (44)$$

$$\alpha_X = 1196.2956\ (1+1.3.\ \rho_X /1000)^{2}\ .\ \rho_X^{-1.04} / \ f_X^{2} \qquad (45)$$

Finally we substitute for f_X from (27):

$$\alpha_X = 1196.2956\ (1+1.3.\rho_X /1000)^2.\rho_X^{-1.04+0.8} / (18.0467^2\ (1 + 1.3\ \rho_X /1000)^2\ ) \qquad (46)$$

$$\boxed{\alpha_X = 3.6732\ \ .\ \rho_X^{-0.24}} \qquad (48)$$

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Energy Ratios from Kamlet & Finger Theory

Equation (48) is identical to equations (14 & 15) of Reference [5] 28th ISB Atlanta, derived from Kamlet & Finger's Gurney theory. Whilst (48) is derived here from the generalised Cooper's & Koch's Gurney theories. Previously the author [5] had defined an Energy constant β so that:

$$\alpha_X = \beta \,.\, \rho_X^{\,-0.24} \qquad (49)$$

Where:

$$\beta = (10^{1.6} .\, 0.887^{2} .1.386 / k)\, z = 2.7864z = \quad 3.6732 \qquad (50)$$

So from three theories:

$$EG_X / EHW_X = \alpha_X = \beta \,.\, \rho_X^{\,-0.24} \qquad (51)$$

$$\boxed{\alpha_X = 3.6732 \,.\, \rho_X^{\,-0.24}} \qquad (52)$$

For TNT with an explosive density (ρ_{TNT}) of 1,640 (kg/m^3), from both (48 & 52) we get:

$$\boxed{\alpha_{TNT} = 0.6216} \qquad (53)$$

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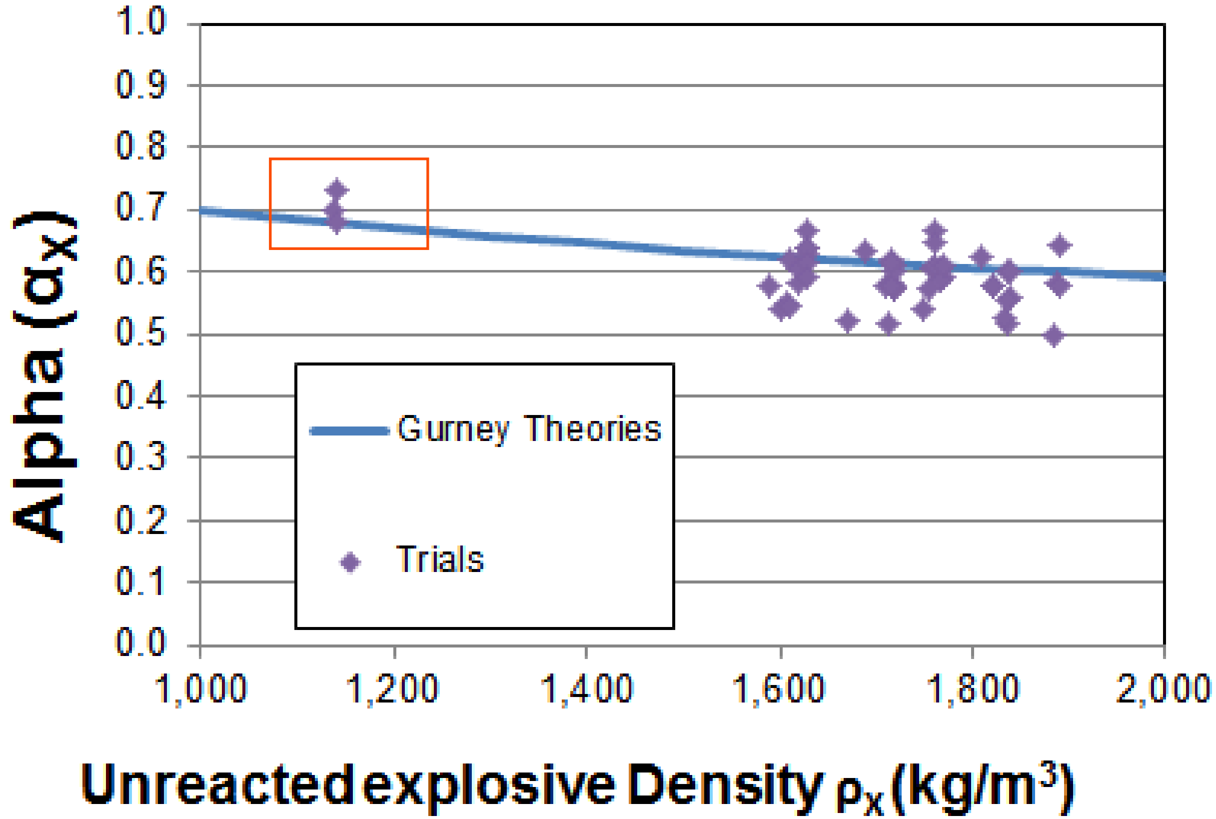

Figure 4. Alpha Sensitivity to Density.

$$\alpha_X = EG_X / EHW_X = \beta / \rho_X^{\,0.24} \qquad (51)$$

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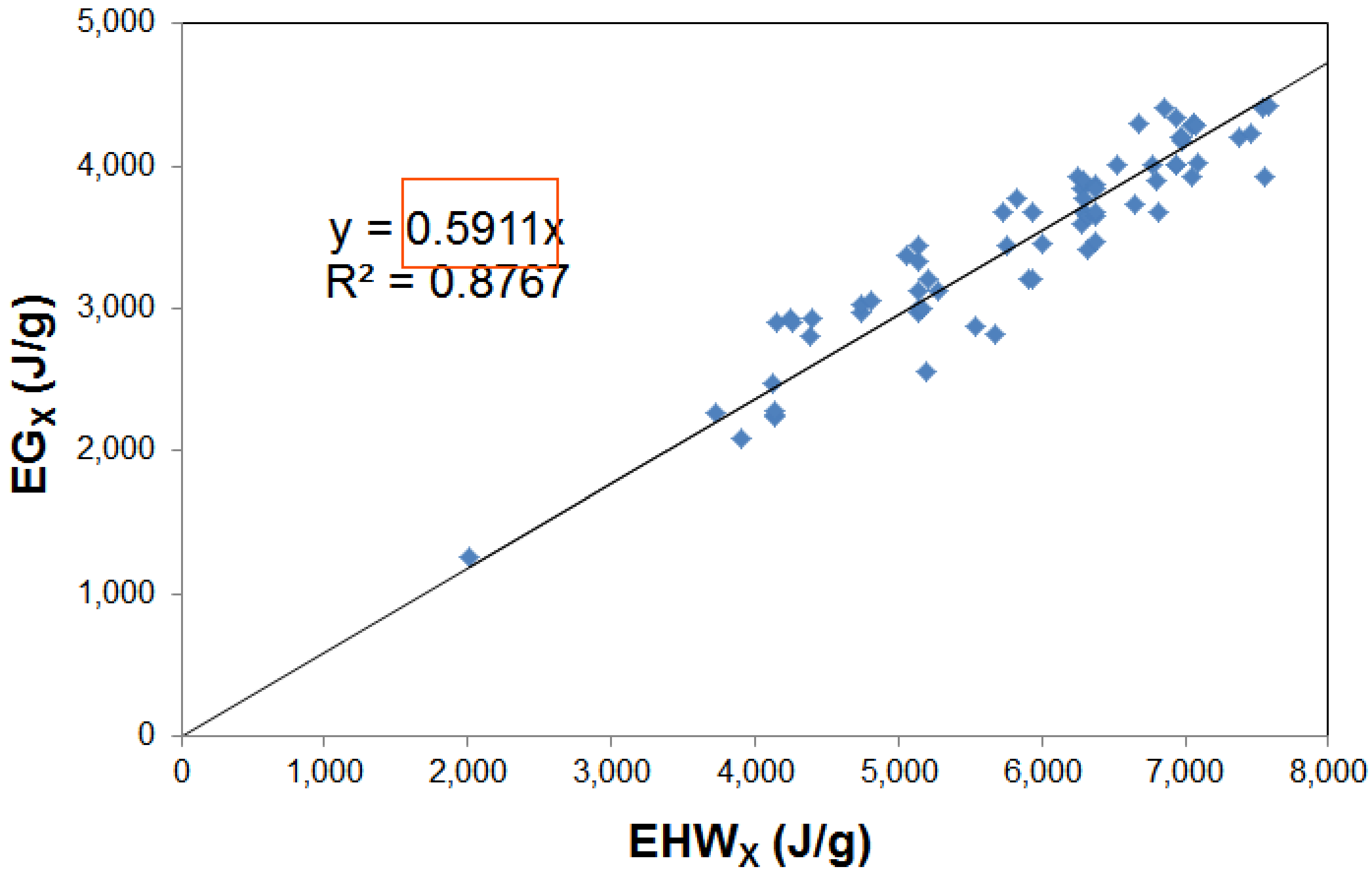

Figure 5. Gurney Energy vs Hydrodynamic Work by Trials.

$$\alpha_{TNT} = EG_{TNT} / EHW_{TNT} = 0.6216 \quad (53)$$

Conclusions #1

- Kamlet & Finger's Gurney Velocity is:

  a) A function of CJ Detonation Pressure and unreacted density

  b) A function of Detonation Velocity and unreacted density

- Hence the CJ Detonation Pressure is a function of Detonation Velocity and unreacted density

Conclusions #2

- Cooper and Koch's Gurney Velocity theories are similar, being both a simple function of the Detonation Velocity divided by a constant
- We can generalise Cooper and Koch's theories by use of a function f_X Where f_X is simply a function of the unreacted explosive density
- The Energy Ratio of Gurney to Hydrodynamic Work here from Cooper and Koch's generalised theory is identical to that previously derived (28th ISB Atlanta) for just Kamlet and Finger's Gurney theory
  - This Ratio is just a function of unreacted explosive density
  - Fragment Energy is typically 62% of the Charge Energy

Conclusions #3

- Recommend varying coefficient (f_X) with unreacted explosive density, instead of using Cooper's original fixed value of 2.97 or Koch's fixed value of 3.08
- Varying coefficient (f_X) gives slightly improved Gurney Velocity predictions, compared with both Cooper's and Koch's theories
  - Approximating the more advanced Gurney theory of Kamlet & Finger
- Koch's fixed coefficient (f_X) of 3.08 is not recommended

Thank you

Any Questions?

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