Assessing System Reliability Growth When Failure Modes Are Masked

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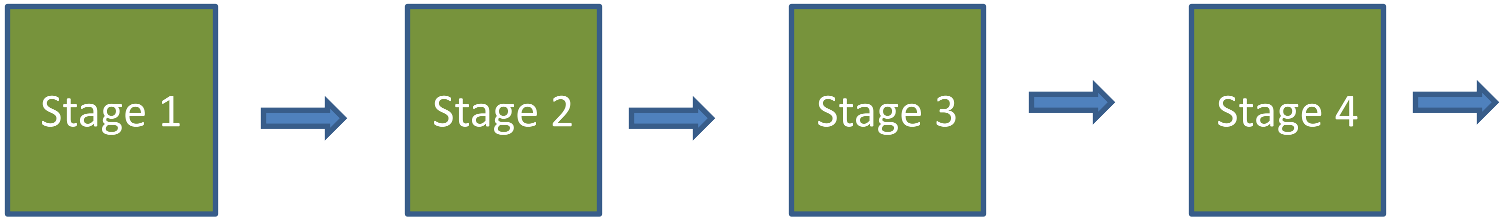
Series System

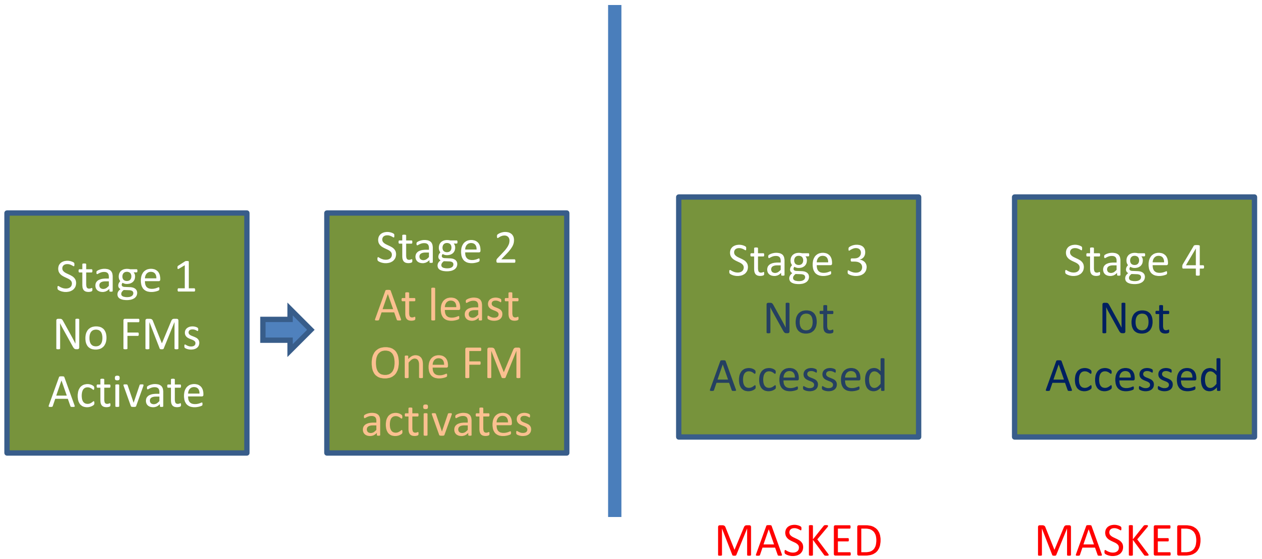


- Random Number of Design Faults/Failure Modes (FMs) in Each Stage/Interface
- When Stage is Accessed Each Remaining FM May Activate Independently of Other FMs with Probabilities Different for Each Stage

Failure Modes (FMs) and Masking

- Each Stage may contain FMs
- If at least one FM activates in stage s then test does not proceed to stages s+1,s+2,…,S
  - The FMs in subsequent stages are **MASKED**
- All activated FMs removed prior to next test

IF No FMs Activate In Stage 1 &
At Least One FM Activates in Stage 2
Stages 3 & 4 Are Masked



$W(t;s)$=Number of Times Stage s is Accessed During Tests 1,2,…,t

- If at least one FM activates in stages 1,2,…,s-1 during test t+1, (Stages s, s+1,…,S MASKED)

$$W(t+1;s)=W(t;s)$$

- If no FMs activate in stages 1,2,…,s-1 during test t+1 (Stage s Accessed)

$$W(t+1;s)=W(t;s)+1$$

Model for Number of Failure Modes (FMs)

- Poisson number FMs, each Stage, prior to testing
  - m(0;s)= mean number FMs, stage s
- FM, stage s, activates with probability p(s) independently of other FMs
  - No masking of FMs within a stage
- If at least one FM activates in stage s then test does not proceed to stages s+1,s+2,…,S
  - FMs in subsequent stages MASKED
- Activated FMs removed prior to next test
  - (To be generalized)

Distribution of FMs Remaining

- Conditional distribution of number of FMs remaining in stage s after accessed W(t;s) times

  Poisson with mean $m(0;s)(1-p(s))^{W(t;s)}$.

- Conditional probability 0 FMs activate in stage s after accessed W(t;s) times

  $\text{Exp}\{-m(0;s)(1-p(s))^{W(t;s)}p(s)\}$

- Independence within/between tests strongly assumed
  - No common cause or shocks (Later!)

Conditional Probability Stage s Accessed on Test t+1 Given W(t;1),…,W(t;s-1)

- Probability 0 FMs Activate in stages 1,2,…,s-1

$$a(t;s-1)=\text{Exp}\{-[A(t;1)+A(t;2)+\ldots+A(t;s-1)]\}$$

where

$$A(t;k)=m(0;k)p(k)(1-p(k))^{W(t;k)}$$

$$k=1,2,\ldots,s-1$$

Simulation for Test t+1

- For each test t+1 generate a uniform random variable on [0,1]: U_1

  $$U_1 \le a(t;s-1) \;\&\; U_1 > a(t;s)$$

  0 FMs activate in Stages 1,2,…,s-1 & at least one s-stage-FM activates on (t+1)th test

  Stages s+1,…,S MASKED

- If 0 FMs activate in Stages 1,2,…,S-1, generate a uniform random variable on [0,1]: U_2

  $$U_2 \le \mathrm{Exp}\{-m(0;S)(1-p(S))^{W(t;S)}p(S)\}$$

  0 FMs are activated in the last stage, S, or before

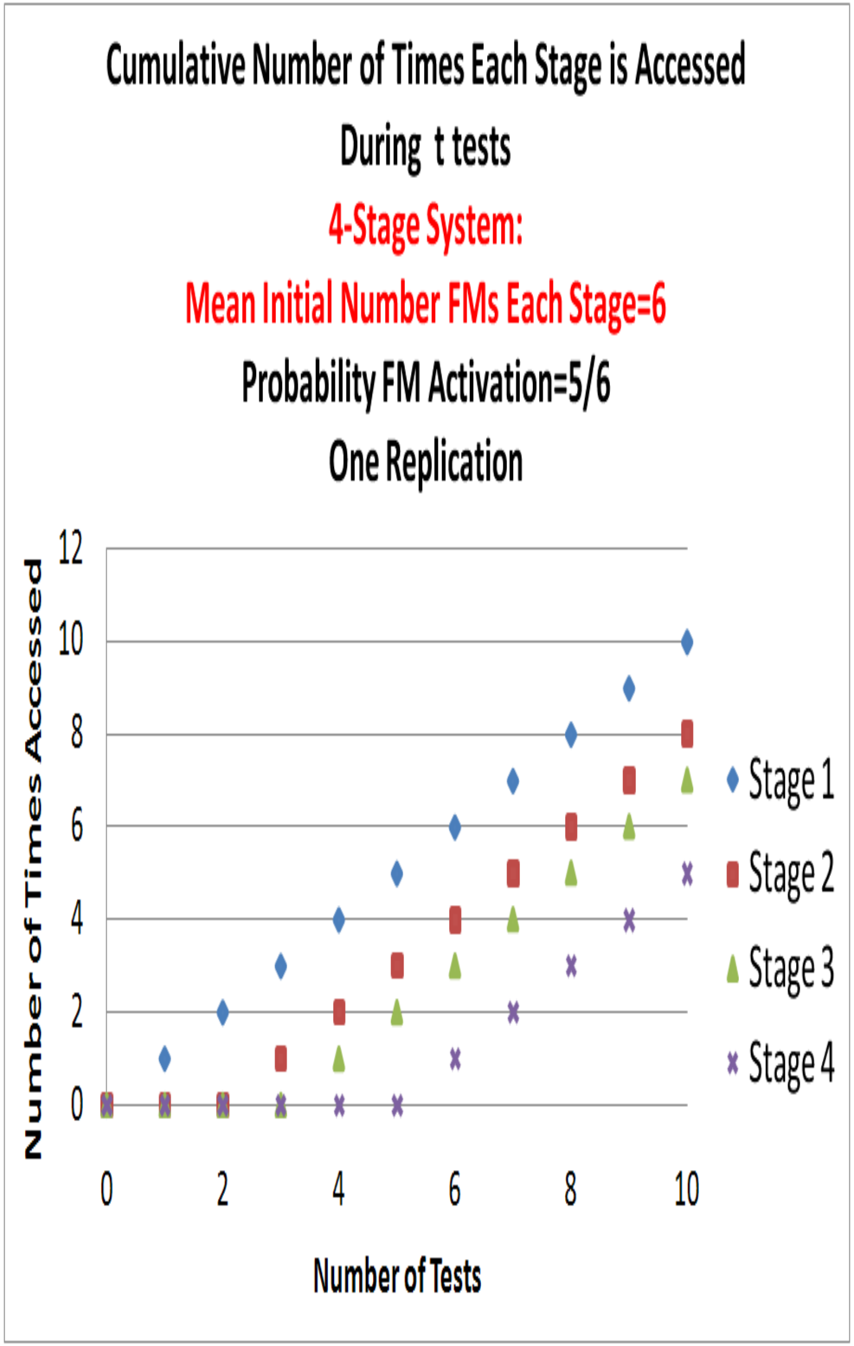
  &

  0 FMs are activated in the entire system on test t+1

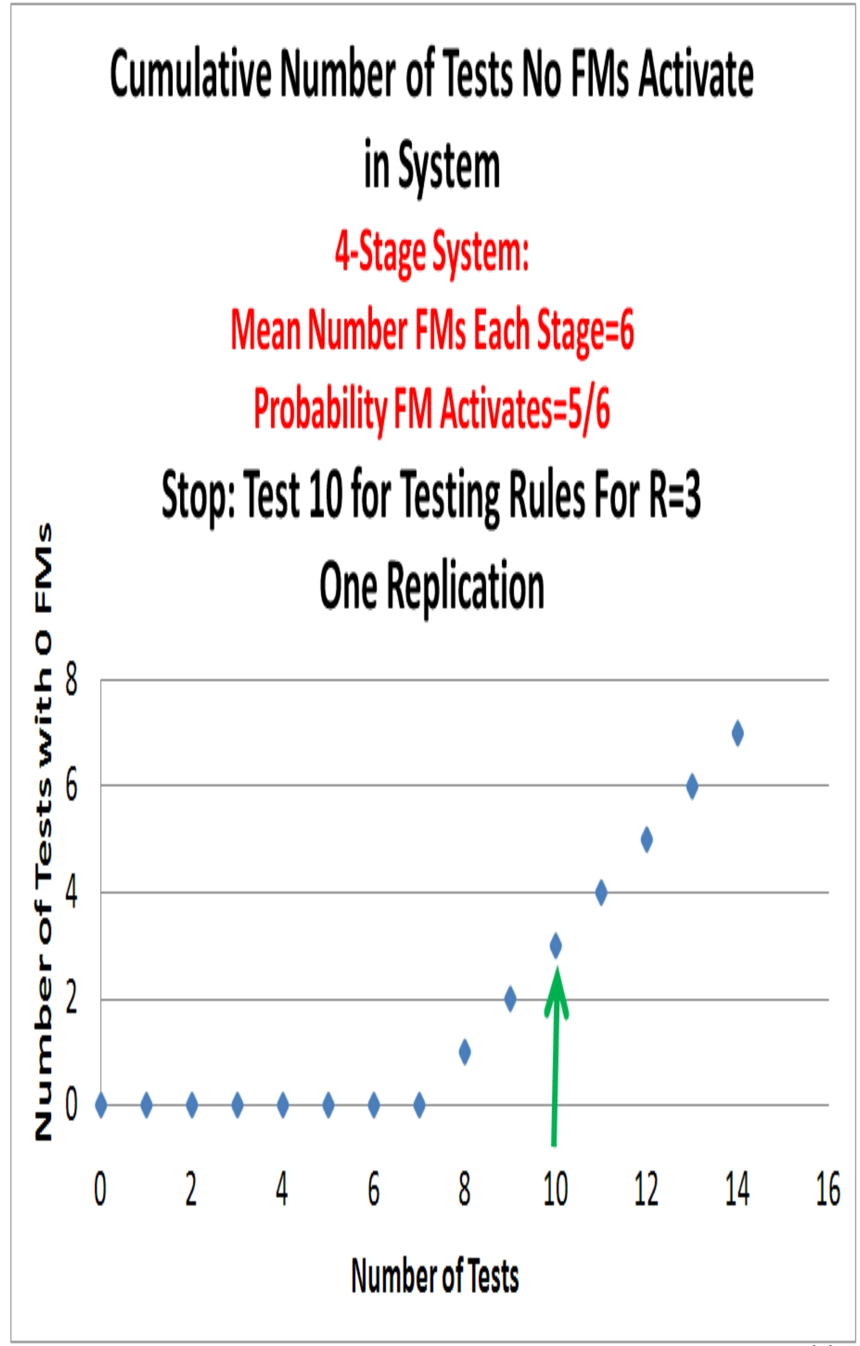
  (Optional: another test)

Stopping Rules

- Test: until 0 FMs activate, all stages, R tests
- Test: until 0 FMs activate, all stages, R consecutive tests
- Fixed Number of Tests
- Common Simulation Replication, Number Times Each Stage is Accessed & Number Times 0 FMs Activate, All Stages



Cumulative Number of Times Each Stage is Accessed
During t tests
4-Stage System:
Mean Initial Number FMs Each Stage=6
Probability FM Activation=5/6
One Replication
Number of Times Accessed
Number of Tests
0
2
4
6
8
10
12
Stage 1
Stage 2
Stage 3
Stage 4



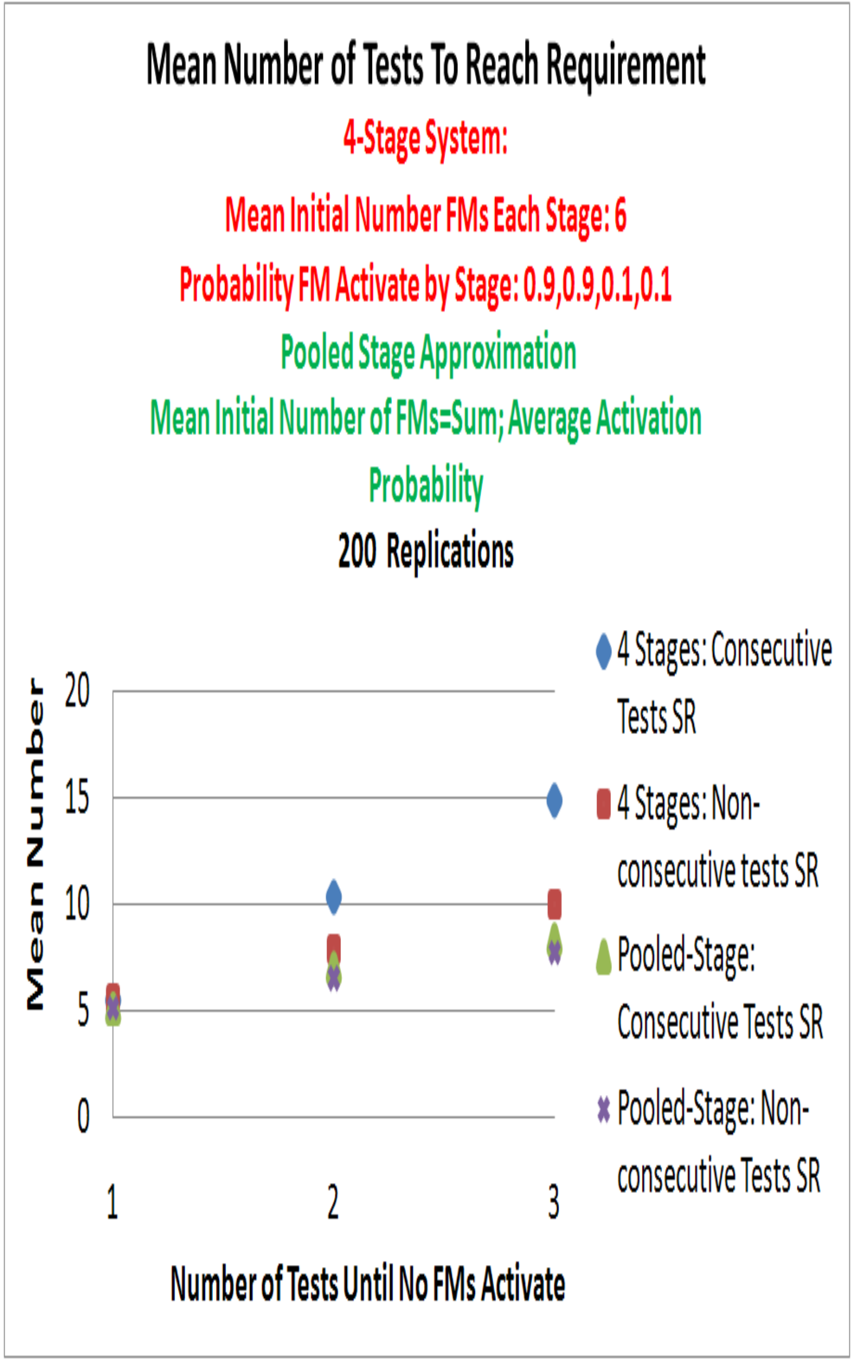
Cumulative Number of Tests No FMs Activate
in System
4-Stage System:
Mean Number FMs Each Stage=6
Probability FM Activates=5/6
Stop: Test 10 for Testing Rules For R=3
One Replication
Number of Tests with 0 FMs
Number of Tests
0
2
4
6
8
10
12
14
16

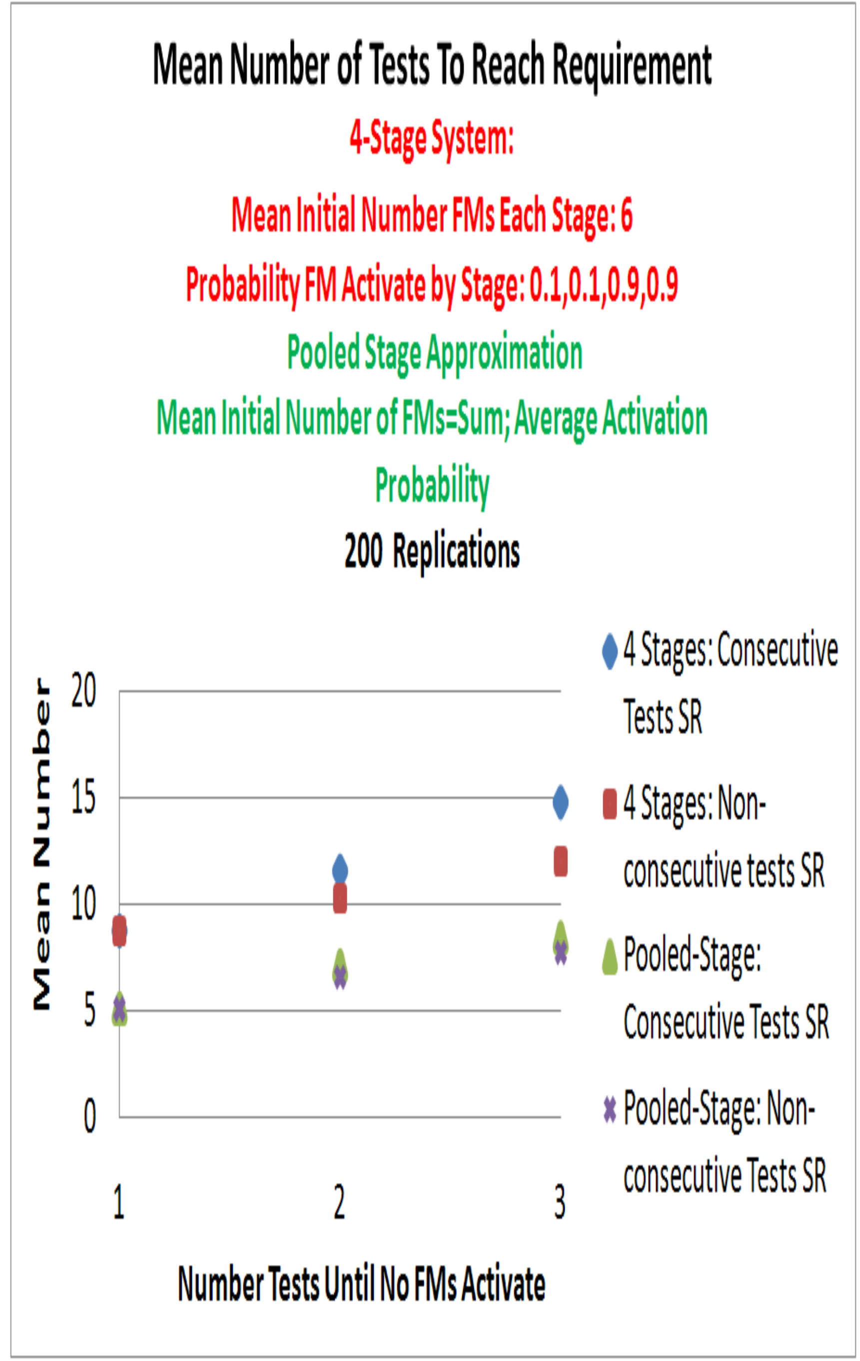
Approximate Pooled 1-Stage System

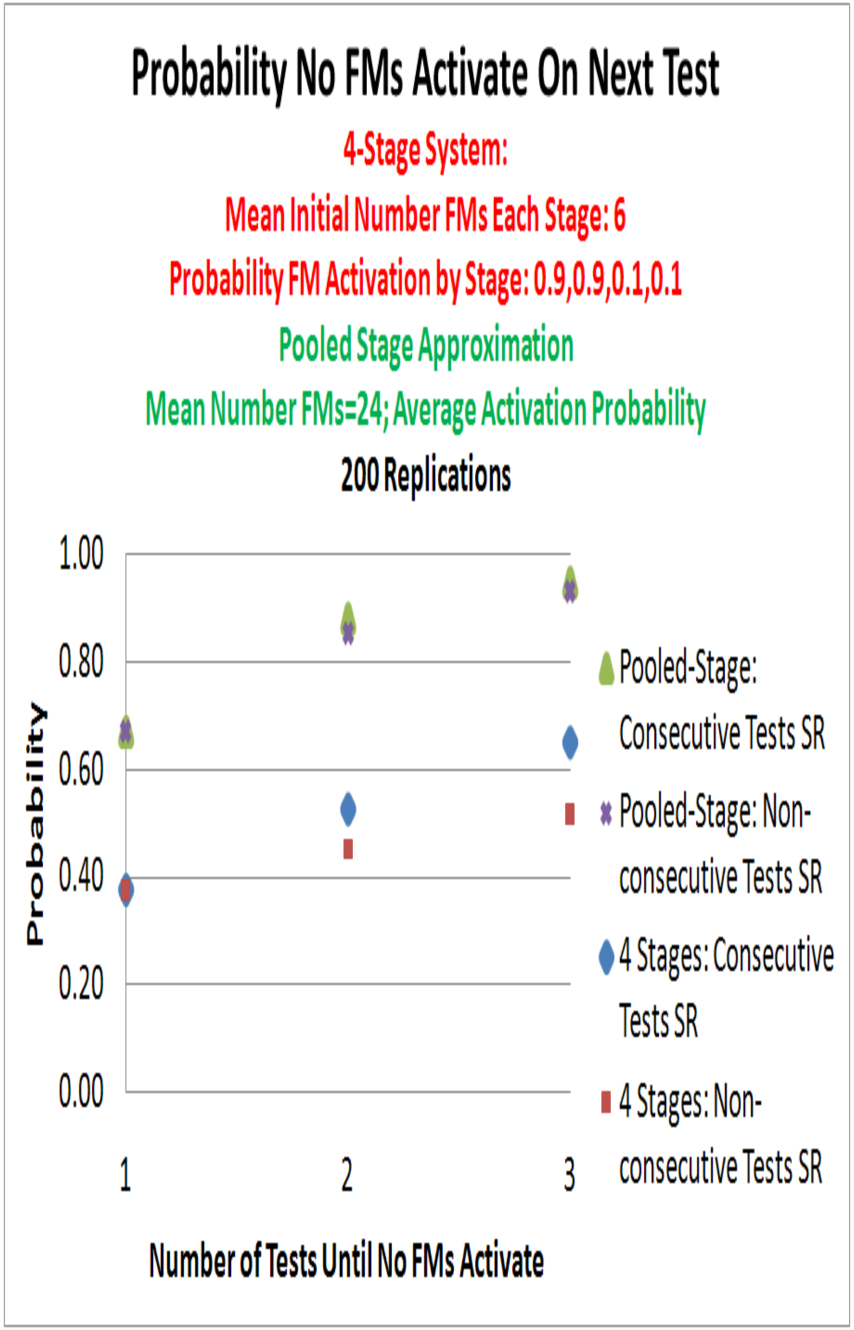
- Initial number FMs has Poisson distribution with mean the sum of the mean FMs in each stage
- Probability a FM activates =p
  - p=sum(m(0;s)p(s))/sum(m(0;s))
- Each Test: All remaining FMs are subject to activation (NO MASKING)

If All Accessed FMs have Same Activation Probability in Both S-Stage and Pooled Systems

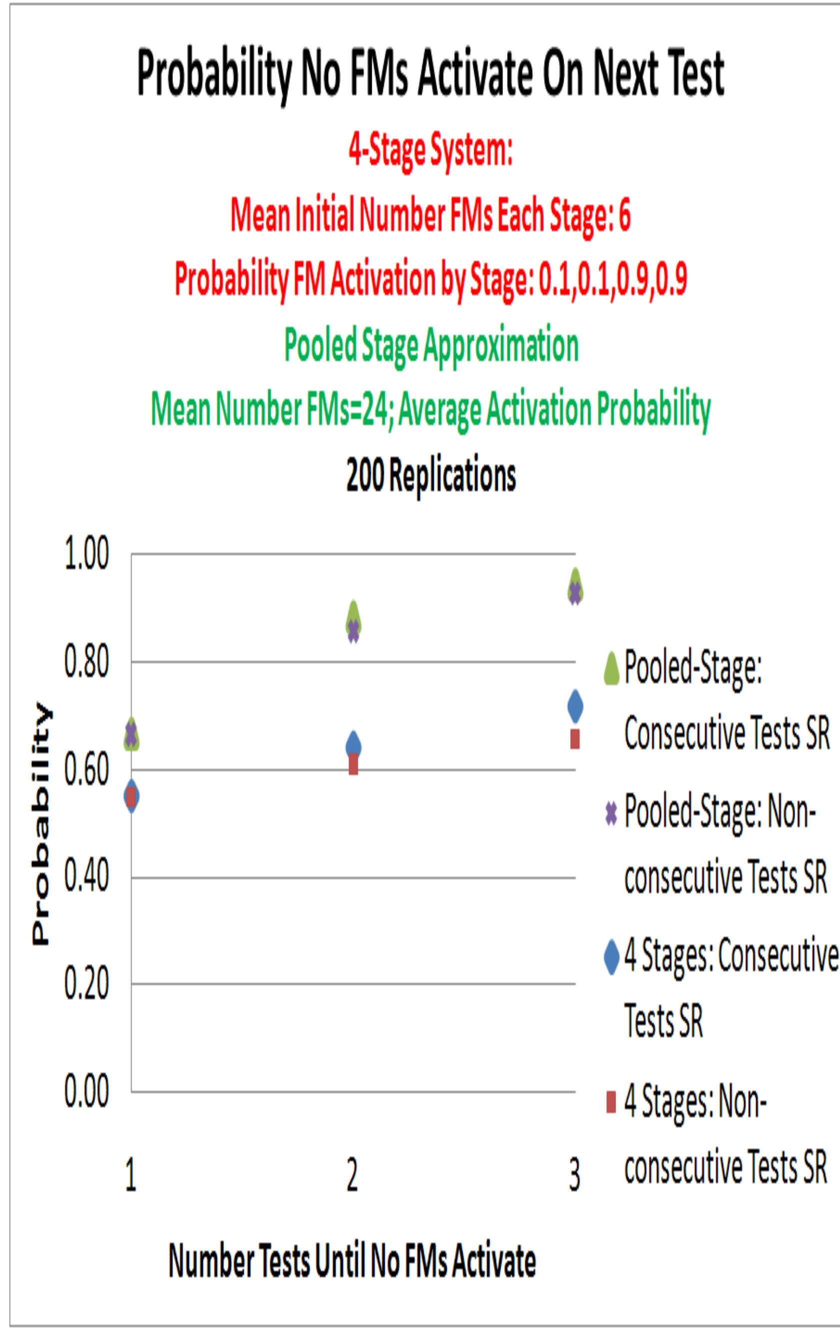
S-Stage System (MASKING)		Pooled 1-Stage System (NO MASKING) OPTIMISTIC
Number of tests until meet stopping criterion	Stochastically $\geq$	Number of tests until meet stopping criterion
Probability 0 FMs activate on one more test after stopping	$\leq$	Probability 0 FMs activate on one more test after stopping


Mean Number of Tests To Reach Requirement
4-Stage System:
Mean Initial Number FMs Each Stage: 6
Probability FM Activate by Stage: 0.9,0.9,0.1,0.1
Pooled Stage Approximation
Mean Initial Number of FMs=Sum; Average Activation Probability
200 Replications
Mean Number
0
5
10
15
20
1
2
3
Number of Tests Until No FMs Activate
4 Stages: Consecutive Tests SR
4 Stages: Non-consecutive tests SR
Pooled-Stage: Consecutive Tests SR
Pooled-Stage: Non-consecutive Tests SR

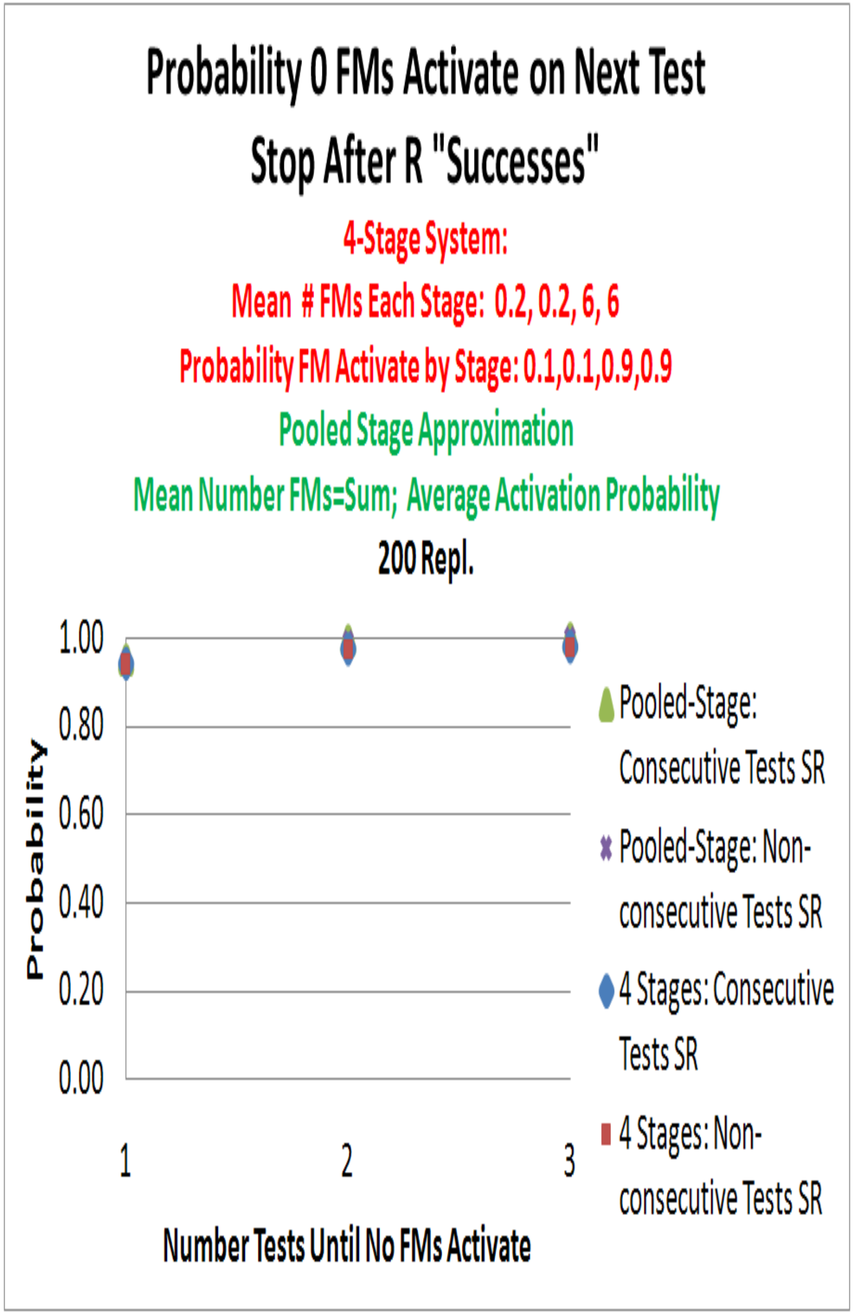

Mean Number of Tests To Reach Requirement
4-Stage System:
Mean Initial Number FMs Each Stage: 6
Probability FM Activate by Stage: 0.1,0.1,0.9,0.9
Pooled Stage Approximation
Mean Initial Number of FMs=Sum; Average Activation Probability
200 Replications
Mean Number
0
5
10
15
20
1
2
3
Number Tests Until No FMs Activate
4 Stages: Consecutive Tests SR
4 Stages: Non-consecutive tests SR
Pooled-Stage: Consecutive Tests SR
Pooled-Stage: Non-consecutive Tests SR



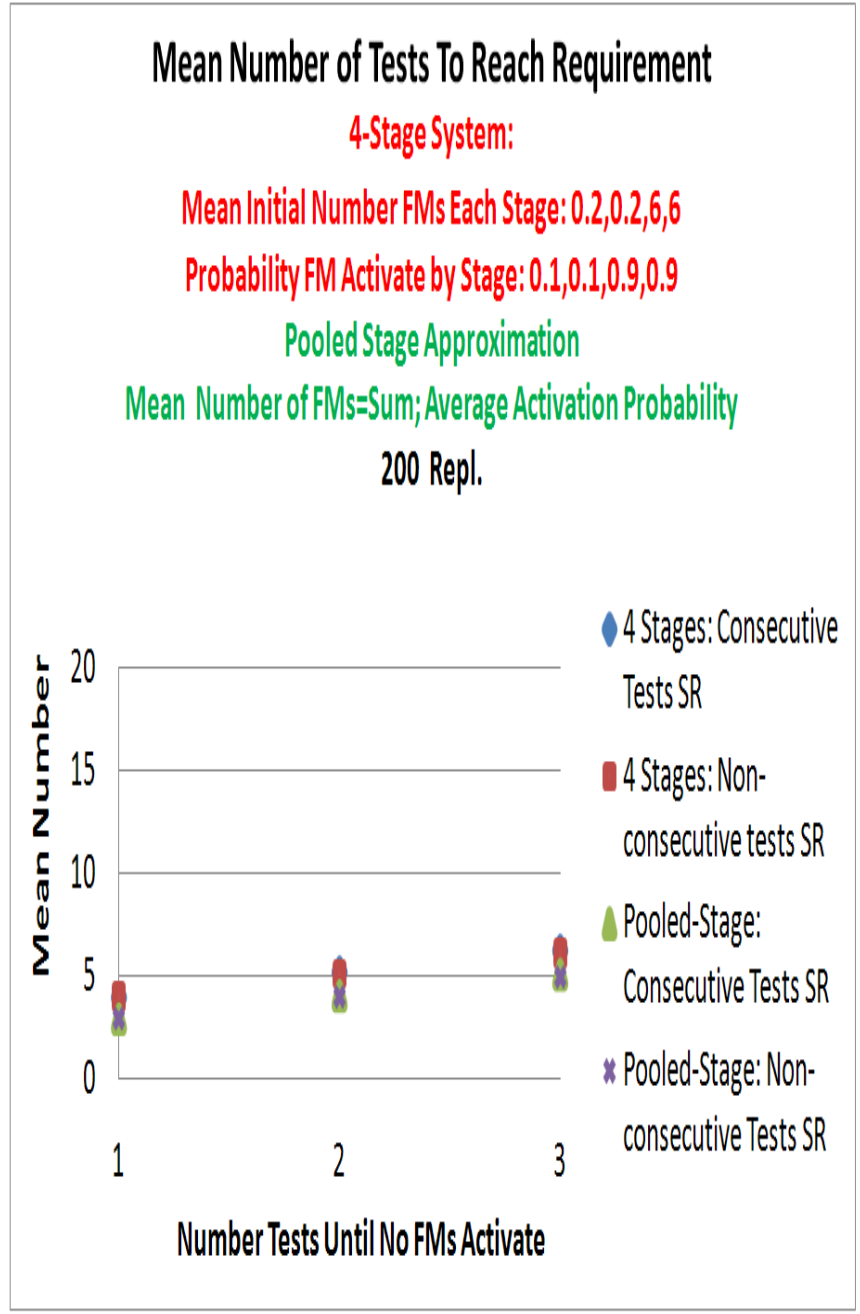
Probability No FMs Activate On Next Test
4-Stage System:
Mean Initial Number FMs Each Stage: 6
Probability FM Activation by Stage: 0.9,0.9,0.1,0.1
Pooled Stage Approximation
Mean Number FMs=24; Average Activation Probability
200 Replications
Probability
1.00
0.80
0.60
0.40
0.20
0.00
1
2
3
Number of Tests Until No FMs Activate
Pooled-Stage: Consecutive Tests SR
Pooled-Stage: Non-consecutive Tests SR
4 Stages: Consecutive Tests SR
4 Stages: Non-consecutive Tests SR



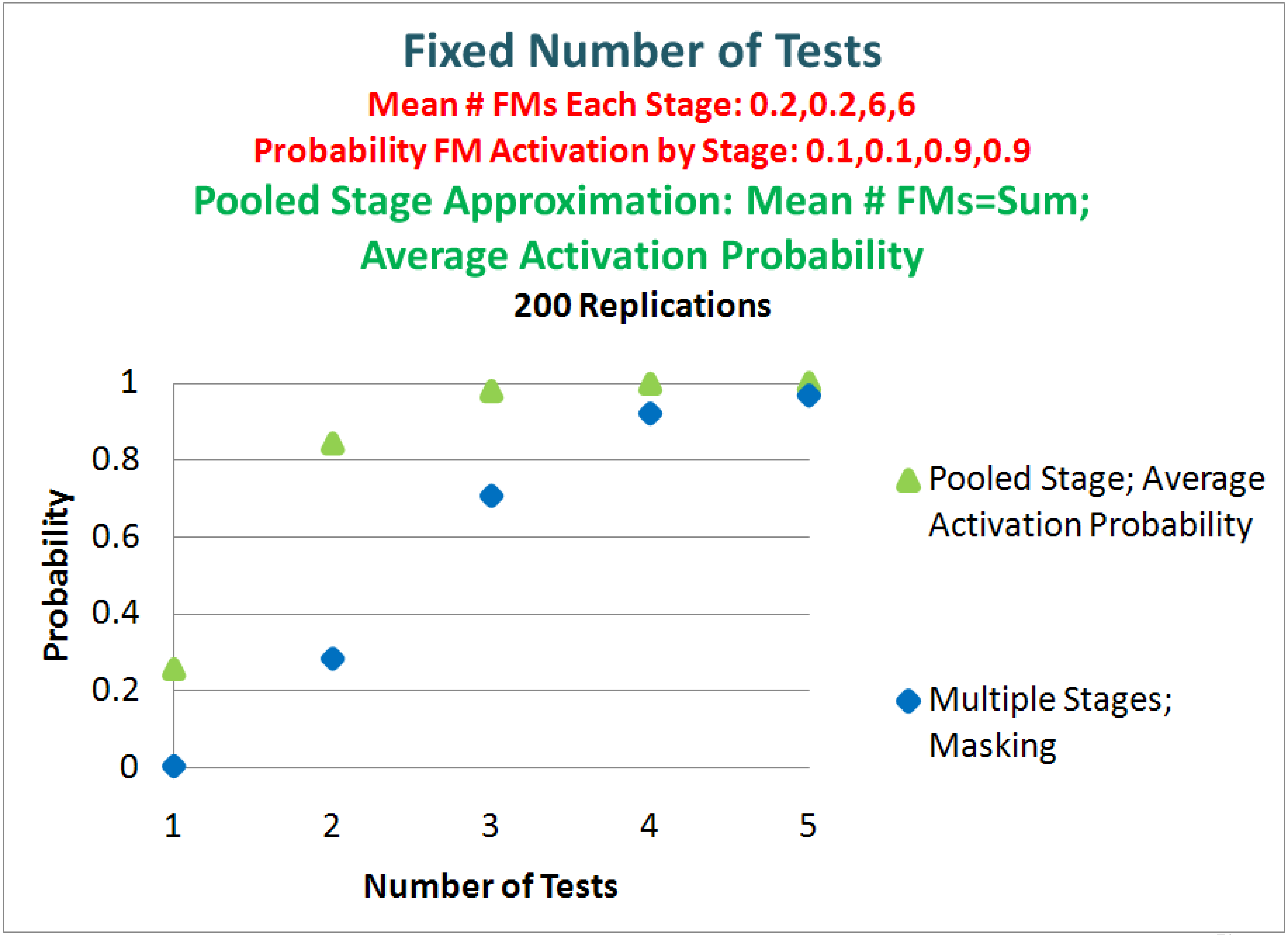
Probability No FMs Activate On Next Test
4-Stage System:
Mean Initial Number FMs Each Stage: 6
Probability FM Activation by Stage: 0.1,0.1,0.9,0.9
Pooled Stage Approximation
Mean Number FMs=24; Average Activation Probability
200 Replications
Probability
1.00
0.80
0.60
0.40
0.20
0.00
1
2
3
Number Tests Until No FMs Activate
Pooled-Stage: Consecutive Tests SR
Pooled-Stage: Non-consecutive Tests SR
4 Stages: Consecutive Tests SR
4 Stages: Non-consecutive Tests SR



Probability 0 FMs Activate on Next Test
Stop After R "Successes"
4-Stage System:
Mean # FMs Each Stage: 0.2, 0.2, 6, 6
Probability FM Activate by Stage: 0.1,0.1,0.9,0.9
Pooled Stage Approximation
Mean Number FMs=Sum; Average Activation Probability
200 Repl.
1.00
0.80
0.60
0.40
0.20
0.00
Probability
1
2
3
Number Tests Until No FMs Activate
Pooled-Stage: Consecutive Tests SR
Pooled-Stage: Non-consecutive Tests SR
4 Stages: Consecutive Tests SR
4 Stages: Non-consecutive Tests SR



Mean Number of Tests To Reach Requirement
4-Stage System:
Mean Initial Number FMs Each Stage: 0.2,0.2,6,6
Probability FM Activate by Stage: 0.1,0.1,0.9,0.9
Pooled Stage Approximation
Mean Number of FMs=Sum; Average Activation Probability
200 Repl.
20
15
10
5
0
Mean Number
1
2
3
Number Tests Until No FMs Activate
4 Stages: Consecutive Tests SR
4 Stages: Non-consecutive tests SR
Pooled-Stage: Consecutive Tests SR
Pooled-Stage: Non-consecutive Tests SR


Fixed Number of Tests
Mean # FMs Each Stage: 0.2,0.2,6,6
Probability FM Activation by Stage: 0.1,0.1,0.9,0.9
Pooled Stage Approximation: Mean # FMs=Sum;
Average Activation Probability
200 Replications
Probability
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
Number of Tests
Pooled Stage; Average Activation Probability
Multiple Stages; Masking

Summary

- The 1-Stage (Pooled) System can be optimistic compared to system with MASKING
  - Smaller Mean Number of Tests Until Obtain the Required Number of Successes
  - Larger Probability, next test activates no FMs, each stopping rule
- R Consecutive Successful tests versus R Successful tests
  - Larger number of tests,

    BUT
  - Larger probability one more test will not activate FM
- Fixed Number of Tests may not be enough
  - Testing to Learn

Reference

- D. P. Gaver, P. A. Jacobs, K. D. Glazebrook, and E. A. Seglie. "Probability models for sequential-stage system reliability growth via failure mode removal". *International Journal of Reliability, Quality and Safety Engineering*, **10** (2003), 15-40.