SOME OPERATIONAL RESEARCH ISSUES IN SYSTEM TESTING

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Definition

Operations/Operational Research applies quantitative scientific methods to assist & enhance decision making

TESTING INPUTS

- ARRIVAL TRAFFIC TYPES
  - SLOWLY TIME-VARIABLE WITHIN DAYS
  - BETWEEN DAY VARIATION
  - SURGES/ “BUNCH ARRIVALS”
  - PRIORITIES
- SERVERS and SERVICE
  - DATA & APPS: CAPACITY (BYTES), PARALLEL?
  - SERVICE TIMES
  - SECURITY
  - DEADLINES/TIMEOUTS (RANDOM)
  - HIERARCHY
- CONGESTION/LOADING
  - QUEUE MANAGEMENT/CONTROL
  - “INITIAL LOADING” BEFORE TEST

SUMMARY OF APPROACH

- PROBLEM: DESIGN ADAPTIVE COST-EFFECTIVE “CLOUD/SOA” COMPUTER (SERVER & DATA-STORAGE & SECURE) SYSTEM
  - LINK LEGACIES
- IMPORTANCE: GROWING DATA ACQUISITION FOR TIMELY DECISIONS:
  - OWN ASSETS
  - OPPONENTS’
  - NEUTRALS (AVOID)
- INEFFICIENT COST & PERFORMANCE UNLESS PRE-PLANNED & TESTED

“BEST” SOLUTION: PRE-MODEL & ANALYSIS

- DEVELOPMENT (ENGINEERING) TEST (DT)
- SUCCESS/INTERIM FAILURE, REPLAY…
  - VIA TEST & EVALUATION (OT&E)
  - MONITOR FIELD EXPERIENCE
    - MODIFY DESIGN (RAM-C & …)

FOCUS:

- Sizing & Adapting
- Scheduling & Operating
- Sustaining & Restoring Service (RAM-C)

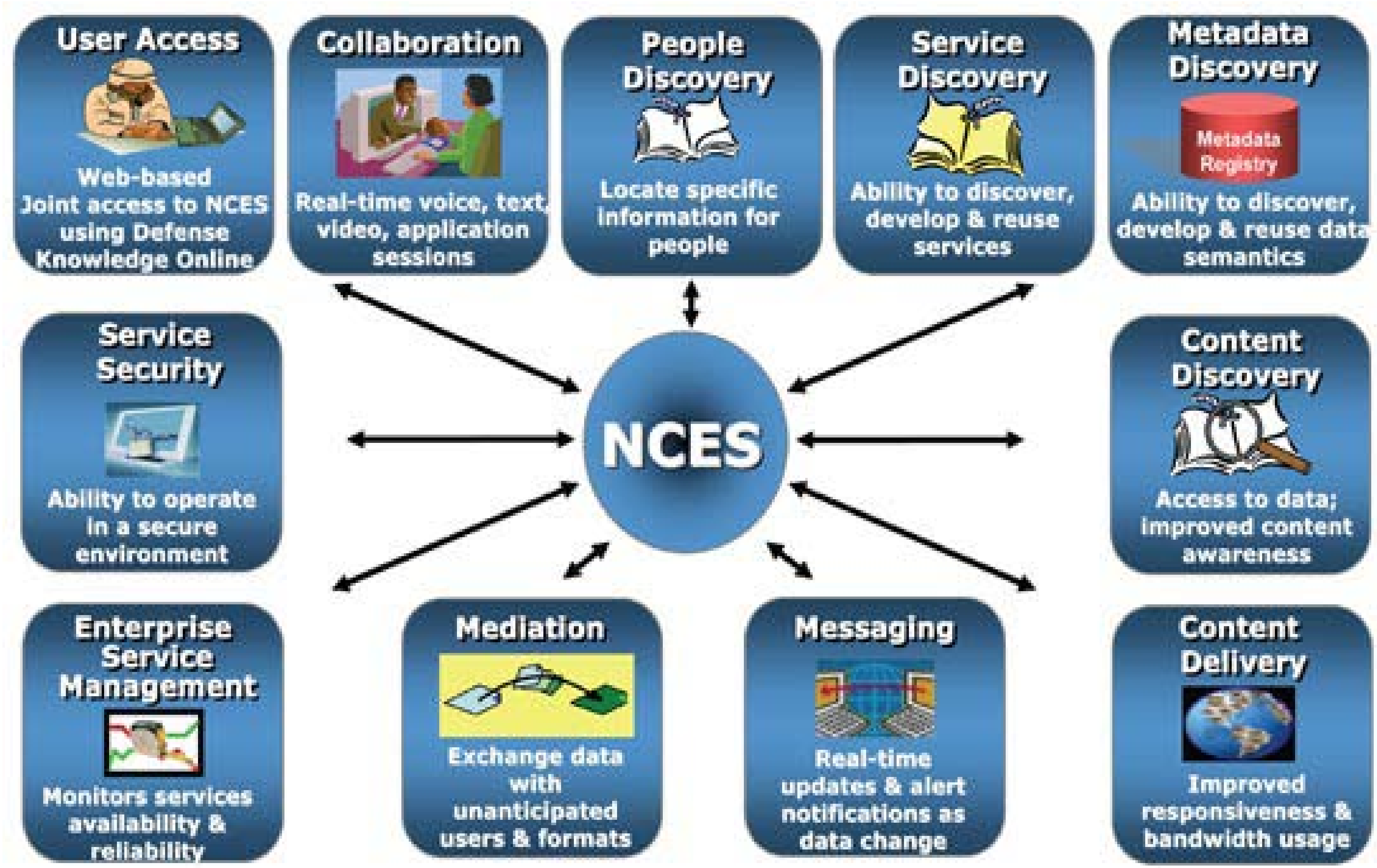
OF

(GENERIC SERVICE-ORIENTED) ARCHITECTURE

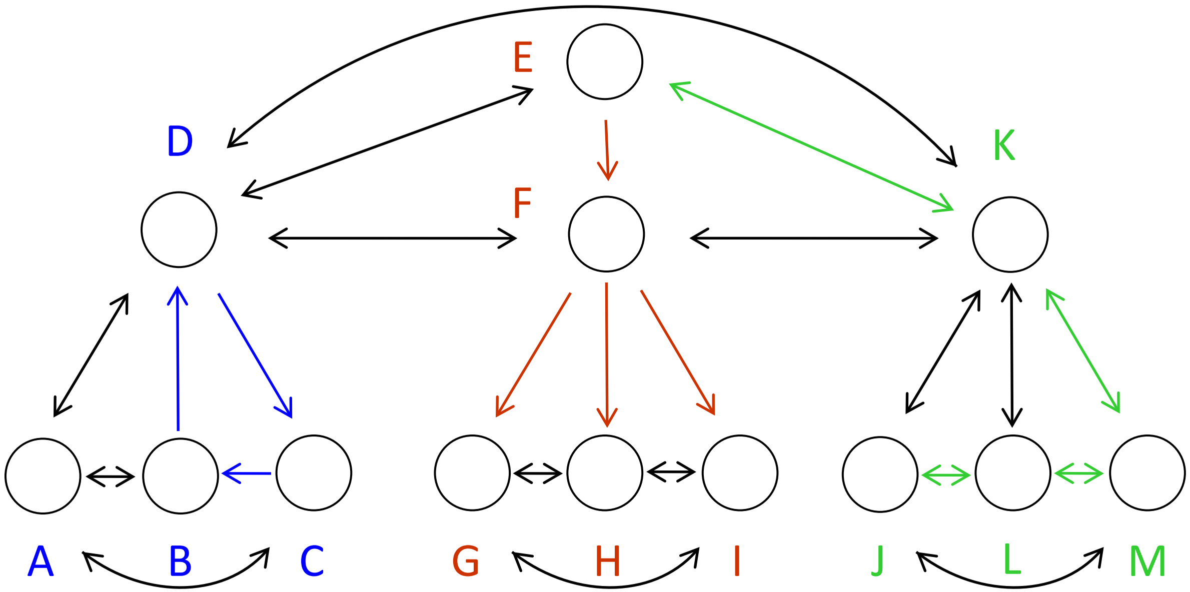
EXAMPLE:

NETWORK CENTRIC ENTERPRISE SERVICES (NCES)

Network Centric Enterprise Services (NCES)



SERVICE-ORIENTED ARCHITECTURE: MINI-CLOUD HIERARCHY

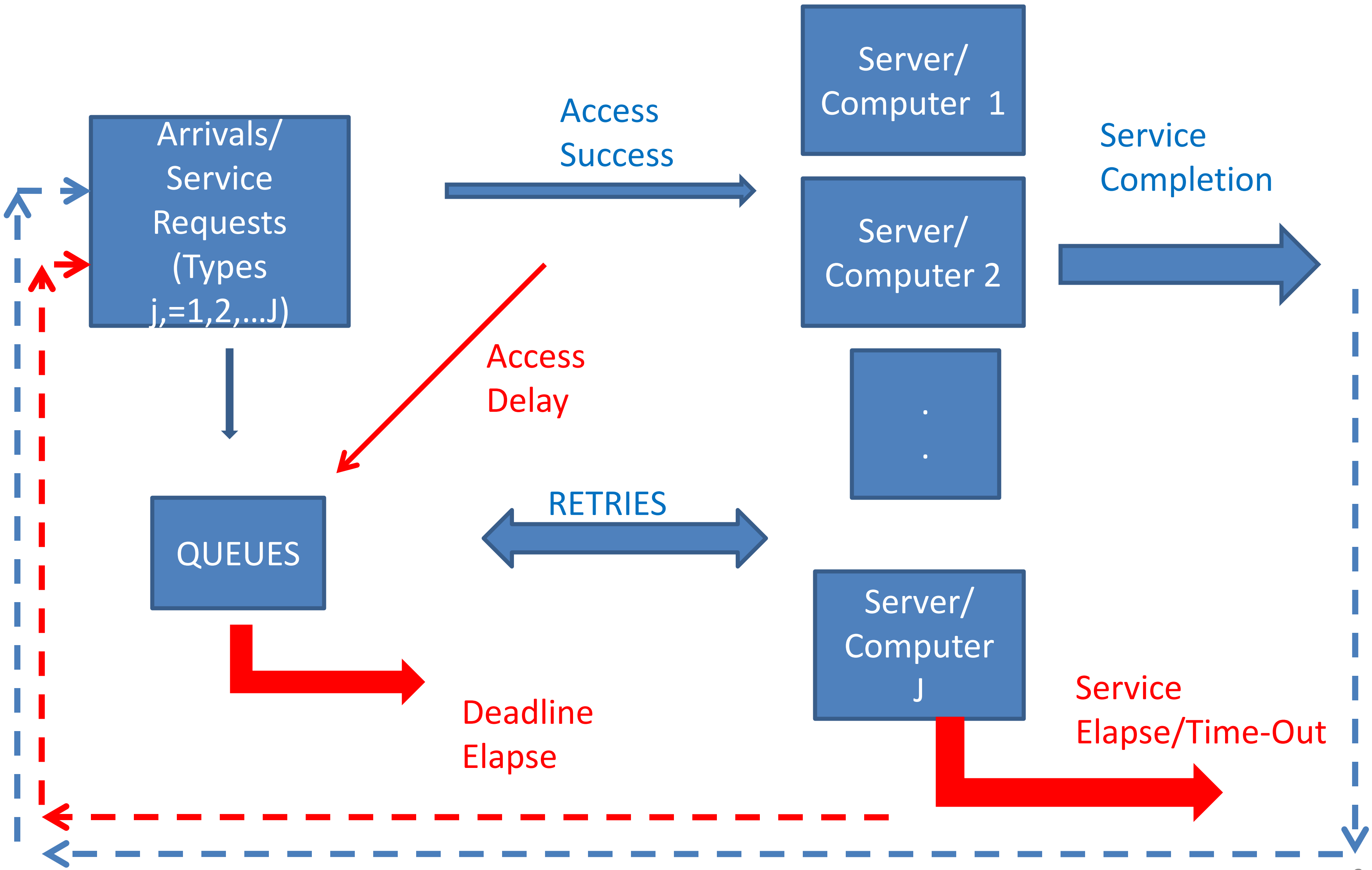


BD: DATA SEARCH, DC: INF. TRANSF, CB: ACK

EF: ORDER/ALERT, FG-FH-FI: BROADCAST TO MINICLOUD

MK: QUERY, KEK: HIGH-LEVEL-RETURN, KM: ACK, ML-LJ: DISTRIBUTE

PROCESS FLOW



Quality of Service (QoS) Issues

- Latency/Delay to Users
- Quality/Accuracy
- Security Quality, Burden to Timeliness
- Cost/Agility/Survivability-Reliability of Infrastructure
- Near Instability/Saturation → Deadline Elapse

IMPLIES

Quantitative Performance Analysis Required

MATH. MODEL

- ARRIVALS/SERVICE TYPE DEMANDS: j=1,2,…,J
- ARRIVALS/SERVICE REQUESTS
  - POISSON, LEVY-KHINCHINE PROCESS:

  RANDOM, RATE: $\lambda_j(t)$, j=1,2,…,J
- SERVICE/COMPUTER STORAGE & TIME
  - STORAGE: $\boldsymbol{C}_j$ (mbytes, flops)
  - SERVICE TIME: $\boldsymbol{S}_j$ (x secs)
    - SERVICE RATE: $\mu_j(t)$

  } RANDOM VARIABLES (INDEPENDENT)
- DEADLINES/DEFECTION TIMES
  - REQUEST SPOILAGE RATE: $\delta_j(t)$
- QUEUE-SERVER:
  - RETRY RATE: $\xi_j(t)$

FLUID MODEL, I (SIMPLEST)

$s_j(t)$ = # TYPE j TASKS IN SERVICE (BY SINGLE COMPUTER)

$q_j(t)$ = # TYPE j TASKS IN QUEUE (VIRTUAL)

$\varphi_j(t)$ = PROB. TYPE j TASK ENTERS SERVICE (BY CLOUD)

$j = 1, 2, \ldots, J$

$$\frac{ds_j(t)}{dt} = \underbrace{\lambda_j \varphi_j(t)}_{\text{Type j task arrives and begins service}} - \underbrace{\left(\mu_j + \delta_j\right) s_j(t)}_{\text{Type j task in service either completes service or its deadline expires}} + \underbrace{\xi_j \varphi_j(t) q_j(t)}_{\text{Type j task in queue retries and begins service}}$$

$$\frac{dq_j(t)}{dt} = \underbrace{\lambda_j \overline{\varphi_j}(t)}_{\text{Type j task arrives, is blocked from beginning service, goes into queue}} - \underbrace{\left[\delta_j + \xi_j \varphi_j(t)\right] q_j(t)}_{\text{Type j task in queue either retries and begins service or its deadline expires}}$$

Fluid Model & Simulation Results I: Model I

$$\lambda_i = 1; C_1 = 1; C_2 = 5; \bar{C} = 100; \frac{1}{\mu_1} = 10; \frac{1}{\mu_2} = 20; \frac{1}{\xi_1} = 10\frac{C_1}{\bar{C}}; \frac{1}{\xi_2} = 20\frac{C_2}{\bar{C}}; \frac{1}{\delta_1} = 20; \frac{1}{\delta_2} = 40$$

	Average of number of type 1 tasks present at arrival times	Average of number of type 2 tasks present at arrival times	Average of number of type 1 tasks in queue present at arrival times	Average of number of type 2 tasks in queue present at arrival times	Fraction of tasks of type 1 lost	Fraction of tasks of type 2 lost
Simulation	6.72 (0.03)	13.2 (0.13)	0.04 (0.008)	0.25 (0.03)	0.33 (0.002)	0.33 (0.001)
Fluid	6.6	12.5	0.23	2.6	0.34	0.38

Fluid & Simulation Results II
Model I

$$\lambda_1 = 2.5; C_1 = 1; C_2 = 5; \bar{C} = 100; \frac{1}{\mu_1} = 10; \frac{1}{\mu_2} = 20; \frac{1}{\xi_1} = 10\frac{C_1}{\bar{C}}; \frac{1}{\xi_2} = 10\frac{C_2}{\bar{C}}; \frac{1}{\delta_1} = 30; \frac{1}{\delta_2} = 60$$

	Average of number of type 1 tasks being served at the arrival times	Average of number of type 2 tasks being served at the arrival times	Average of number of type 1 tasks in queue at the arrival times	Average of number of type 2 tasks in queue at the arrival times	Fraction of tasks of type 1 lost	Fraction of tasks of type 2 lost
Simulation	17.35 (0.12)	16.44 (0.02)	5.43 (0.09)	82.9 (0.91)	0.31 (0.003)	0.66 (0.003)
Fluid	17.9	15.2	4.4	89.4	0.30	0.70

FLUID MODEL, III
PROCESSOR-SHARED COMPUTING

$$\frac{da_j(t)}{dt} = \underbrace{\left[\lambda_j + \xi_j q_j(t)\right]\varphi_j(t)}_{\text{Type j task arrives "new" begins service; or Retry from Queue enters service}} - \underbrace{\overline{C_b}\,\overline{S}\mu_j \frac{g_j a_j(t)}{\sum_{k=1}^{J} g_k a_k(t)} a_j(t)}_{\text{Processor-shared service terminates } (g_j = \text{Static Priority Control})} - \underbrace{\delta_j a_j(t)}_{\text{In-service deadline elapses}}$$

$$\frac{dq_j(t)}{dt} = \underbrace{\lambda p_j \overline{\varphi_j}(t)}_{\text{New arrival enters Queue}} - \underbrace{\left[\delta_j + \xi_j \varphi_j(t)\right] q_j(t)}_{\text{Type j defects from Queue; or Type j Retry enters service}}$$

$$\overline{\varphi_j}(t) = \left[\min\left(\frac{\sum_{k=1}^{J} C_k \left[\frac{a_k(t)}{\overline{S}}\right] + C_j}{\overline{C_b}}, 1\right)\right]^{\kappa}$$

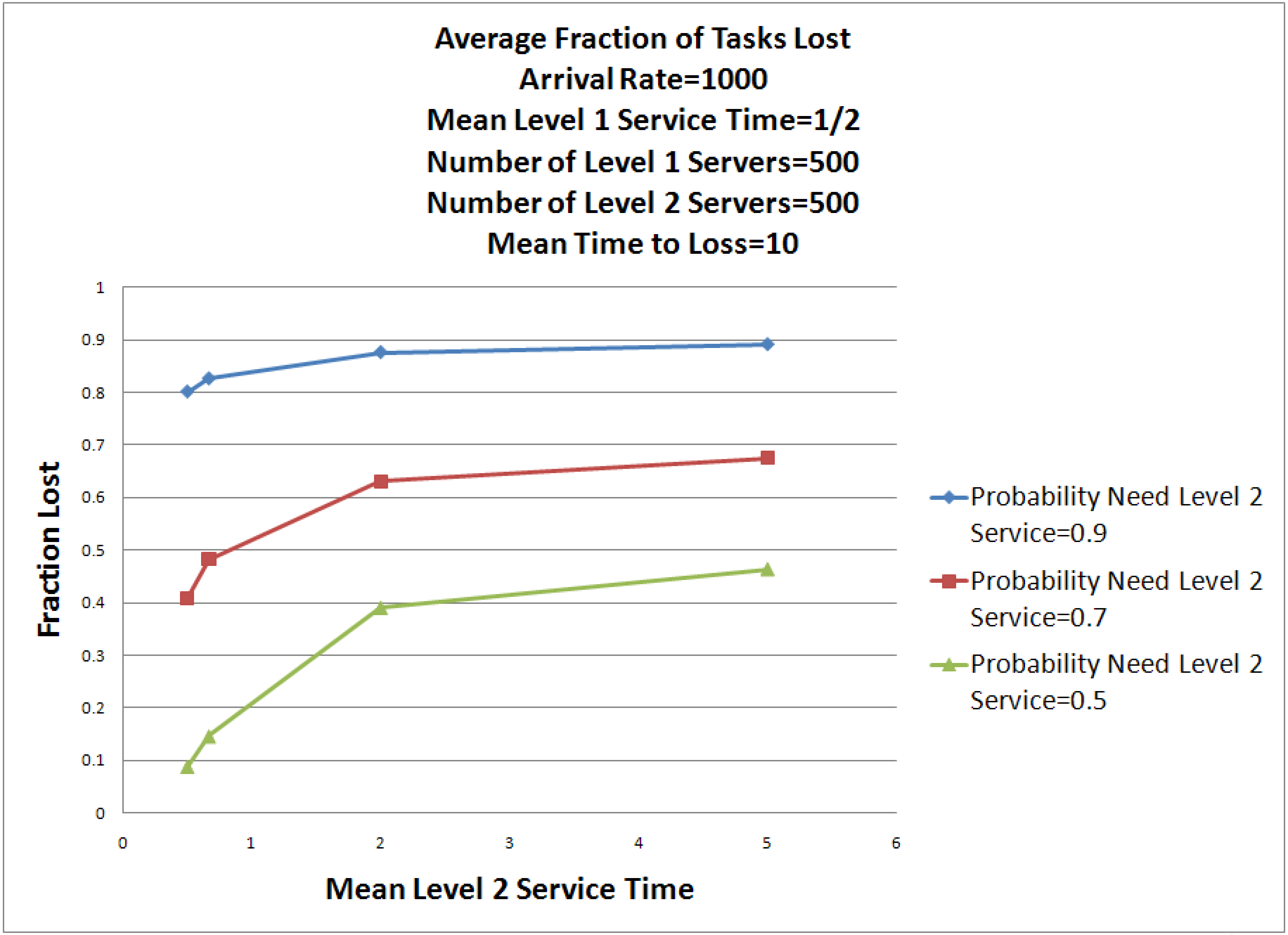
Large Computers are Faster =Fewer Losses

	# of computers in cloud	Total capacity of one computer	Arrival rate of tasks per computer	Probability arriving task is of type 1	Fraction of tasks of type 1 that are **lost**	Fraction of tasks of type 2 that are **lost**	Fraction of tasks that are **lost**
“FCFS”	100	10^2	2×10^3	0.25	0.99	0.99	0.99
“FCFS”	1	10^4	2×10^5	0.25	0.70	0.51	0.56
PS	100	10^2	2×10^3	0.25	0.16	0.37	0.32
PS	1	10^4	2×10^5	0.25	0.12	0.06	0.08

TWO LEVELS OF SERVICE

$$\underbrace{\frac{dn_1(t)}{dt}}_{\substack{\text{Change}\\ \text{in Number}\\ \text{of Tasks}\\ \text{Waiting for}\\ \text{or Being Served}\\ \text{at Level 1}}} = \left[\underbrace{\lambda}_{Arrival} + \underbrace{\mu_2 \min\left(n_2(t), S_2\right)}_{\substack{complete\\ \text{level 2}\\ \text{service}}}\right] - \underbrace{\mu_1 \min\left(n_1(t), S_1\right)}_{\substack{\text{Complete}\\ \text{service}}} - \underbrace{\xi_1 \min\left(n_1(t), S_1\right)}_{\substack{Need\\ \text{Level 2}\\ \text{service}}} - \underbrace{\delta n_1(t)}_{Loss}$$

$$\underbrace{\frac{dn_2(t)}{dt}}_{\substack{\text{Change}\\ \text{in Number}\\ \text{of Tasks}\\ \text{Waiting}\\ \text{for or Being}\\ \text{Served at}\\ \text{Level 2}}} = \left[\underbrace{\xi_1 \min\left(n_1(t), S_1\right)}_{\substack{Need\\ \text{Level 2}\\ \text{service}}}\right] - \underbrace{\mu_2 \min\left(n_2(t), S_2\right)}_{\substack{\text{Complete}\\ \text{service}}} - \underbrace{\delta n_2(t)}_{Loss}$$



Average Fraction of Tasks Lost
Arrival Rate=1000
Mean Level 1 Service Time=1/2
Number of Level 1 Servers=500
Number of Level 2 Servers=500
Mean Time to Loss=10
Fraction Lost
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
Mean Level 2 Service Time
Probability Need Level 2 Service=0.9
Probability Need Level 2 Service=0.7
Probability Need Level 2 Service=0.5

TESTING FOR SYSTEM RELIABILITY GROWTH

- RELIABILITY GROWTH=
  - DESIGN FAULT REMOVAL, MITIGATION
- APPROACH
  - (A) TEST GROUPS OF SUBSYTEMS ("WITHIN")
  - (B) ASSEMBLED GROUPS ("BETWEEN")
- (A)+(B) UNDER TOTAL BUDGET CONSTRAINT, C

ANALYSIS APPROACH

(1) SPLIT SUBSYSTEMS INTO "NATURAL" SUBGROUPS (EACH PLATF. IN SQUADRON, PROPULSION, NAVIGATION, STEERING, ETC)

(2) s_i=NBR ELEMENTS, SUBGROUP i

(3) $s_i+(s_i(s_i-1)/2)$=NBR. INDIV. + PAIRW. TESTS (FIRST/SINGLE) TOTAL TESTS of i

(4) c_i=COST/TEST SUBGROUP i

(5) TOTAL COST SUBGROUP i=$[s_i c_{1i}+(s_i(s_i-1)/2)c_{2i}]$

(6) TOTAL COST, ONE TEST =$\Sigma_i\ [s_i c_{1i}+(s_i(s_i-1)/2)c_{2i}]$

RESULT

$$\sum\left[s_i\theta c_{1i}+\frac{s_i^2\theta^2}{2}c_{2i}\right]-\tilde{C}=0$$

$$\theta_{opt}=\frac{\sqrt{\left(\sum_{i=1}^{I}s_ic_{1i}\right)^2+4\tilde{C}\sum_{i=1}^{I}\frac{s_i^2}{2}c_{2i}}-\sum_{i=1}^{I}s_ic_{1i}}{\sum_{i=1}^{I}s_i^2c_{2i}}$$