Trajectory Deflection of Fin- and Spin-Stabilized Projectiles Using Paired Lateral Impulses

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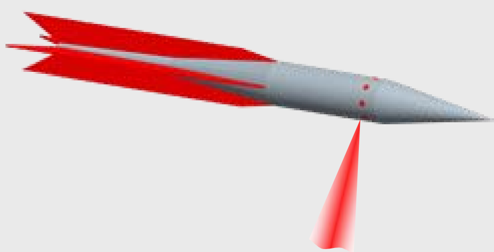
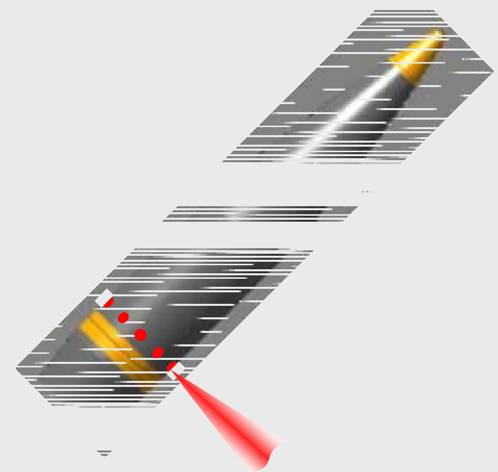
Presentation Overview

- Objective
- Effect of a Lateral Impulse
- Basic Mathematical Model and Analytical Solution
- Pairing the Impulses
- Examples
- Conclusions

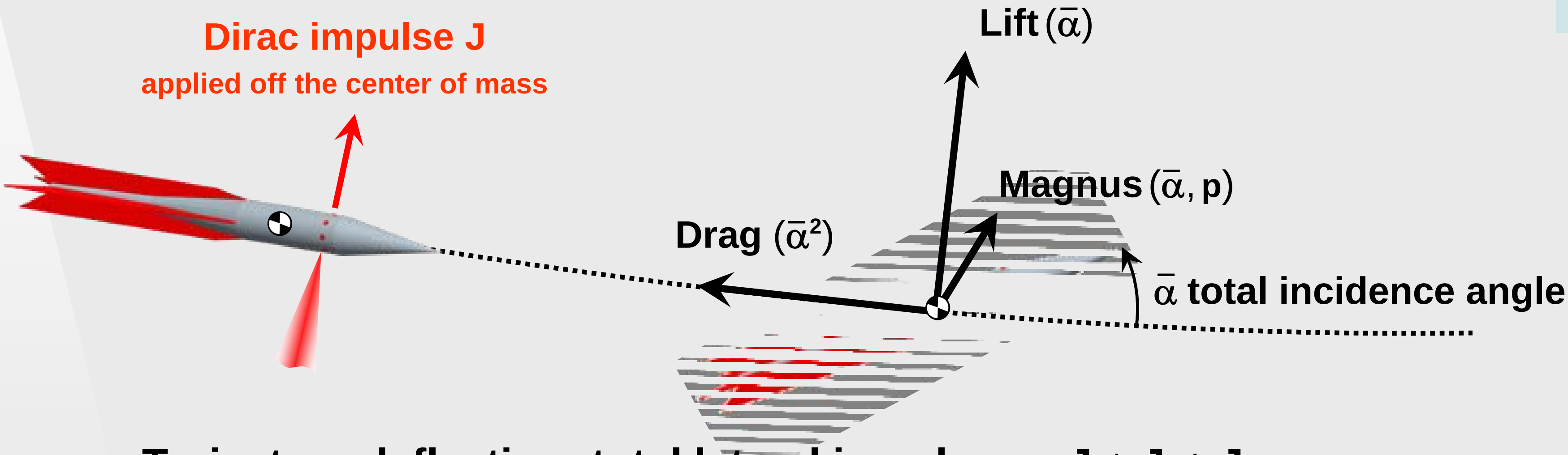
Objective

- **To developed a well defined procedure for using paired impulses on fin- and spin-stabilized projectiles in order to achieve enhanced drift corrections**





Effect of a Lateral Impulse (1/2)



Trajectory deflection: total lateral impulse $= J + J_L + J_Y$

- J_Y → $\int$ Magnus $(\bar{\alpha})$ dt
- J_L → $\int$ Lift $(\bar{\alpha})$ dt

Additional velocity decrease: axial impulse $= J_{D2}$

- J_{D2} → $\int$ Drag $(\bar{\alpha}^2)$ dt

More challenging to implement properly

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Effect of a Lateral Impulse (2/2)

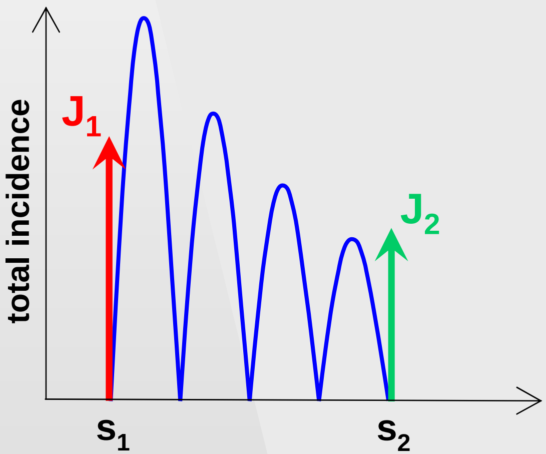
Pairing lateral impulses off the center of mass

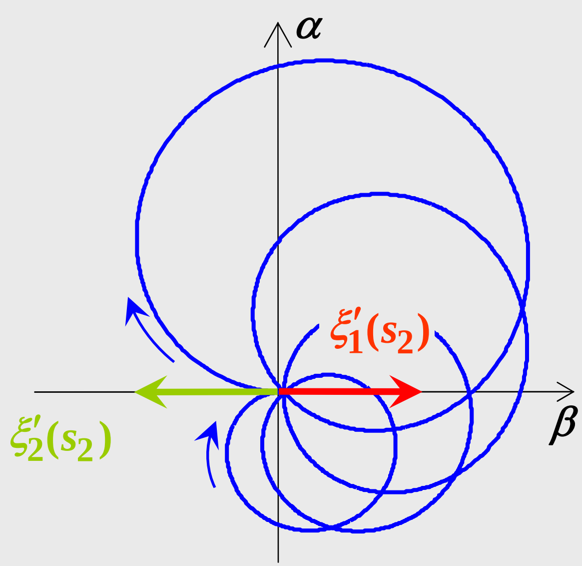
Basic idea: - J_1 triggers the angular motion at s_1

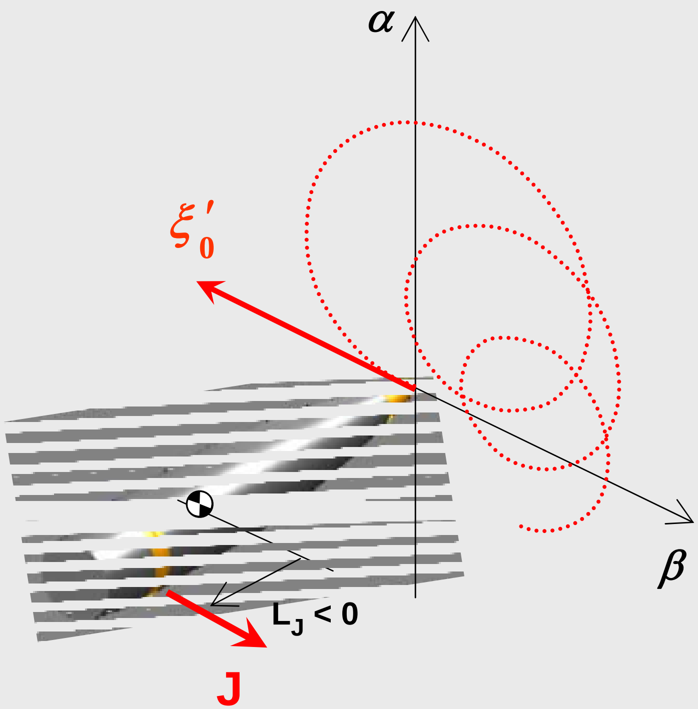
- J_2 stops the angular motion at s_2

Advantages: - the lateral correction is enhanced by the lift impulse resulting from the angular motion

- the range lost is minimized by stopping the angular motion







Basic Mathematical Model (1/4)

Linearized equation of the complex incidence motion:

$$\xi'' + (H - iP)\xi' - (M + iPT)\xi = \xi_0' \delta(s)$$

Dirac impulse

Magnus moment
+ vel. change

$$T = \frac{\rho A d}{2m}\left(C_{L\alpha} + \frac{md^2}{I_x} C_{np\alpha}\right)$$

Overturning moment
(restoring or destabilizing)

$$M = \frac{\rho A d^3}{2 I_y} C_{m\alpha}$$

Gyroscopic effect $P = \frac{I_x}{I_y} \frac{pd}{V}$

Damping factor
+ vel. change

$$H = \frac{\rho A d}{2m}\left(C_{L\alpha} - C_D - \frac{md^2}{I_y} C_{mq}\right)$$

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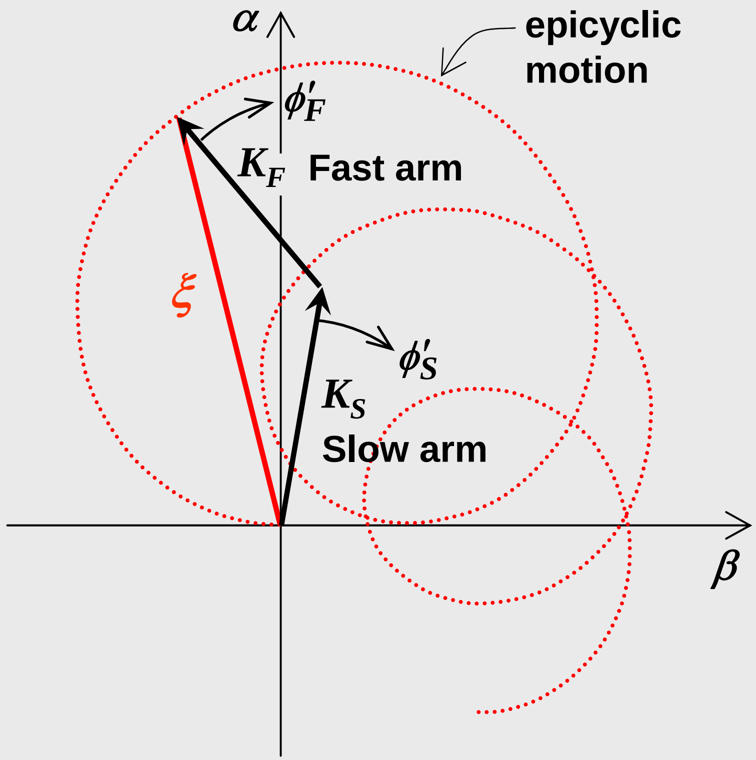
Basic Mathematical Model (2/4)

Motion = sum of two rotating arms:

$$\xi = K_{F_0} e^{i\phi_{F0}} e^{(\lambda_F + i\phi'_F)s} + K_{S_0} e^{i\phi_{S0}} e^{(\lambda_S + i\phi'_S)s}$$



Angular frequencies

$$\phi'_F = \frac{1}{2}\left(P + \sqrt{P^2 - 4M}\right) \qquad \phi'_S = \frac{1}{2}\left(P - \sqrt{P^2 - 4M}\right)$$

Damping factors

$$\lambda_F = -\frac{1}{2}\left(H - \frac{P(2T - H)}{\sqrt{P^2 - 4M}}\right) \qquad \lambda_S = -\frac{1}{2}\left(H + \frac{P(2T - H)}{\sqrt{P^2 - 4M}}\right)$$

Initial arms

$$K_{F_0} e^{i\phi_{F0}} = -\frac{i\xi'_0 + \phi'_S \xi_0}{\phi'_F - \phi'_S} \qquad K_{S_0} e^{i\phi_{S0}} = \frac{i\xi'_0 + \phi'_F \xi_0}{\phi'_F - \phi'_S}$$

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Basic Mathematical Model (3/4)

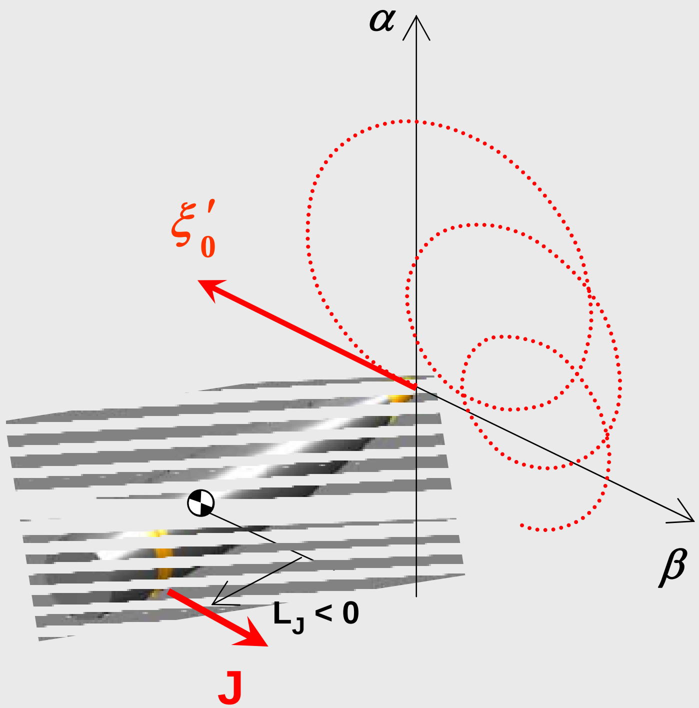
Initial conditions forced by the Dirac impulse:

$$\xi_0 = -\frac{J}{mV}$$

→ **negligible in supersonic mode**

$$\xi_0' = \frac{J\, L_J\, d^2}{I_y V}$$

→ **main cause of angular motion (unless $L_J \to 0$)**



Basic Mathematical Model (4/4)

Lateral impulses: Lift + Magnus

$$J_L = \int_0^\infty L(t)\,dt \qquad J_L = \frac{1}{2}\rho A V d C_{L\alpha}\int_0^\infty \xi\, ds \qquad \int_0^\infty \xi\, ds = -\frac{K_{F_0} e^{i\phi_{F_0}}}{\lambda_F + i\phi'_F} - \frac{K_{S_0} e^{i\phi_{S_0}}}{\lambda_S + i\phi'_S}$$

$$J_Y = \int_0^\infty Y(t)\,dt \qquad J_Y = i\,\frac{C_{Yp\alpha}}{C_{L\alpha}}\frac{p\,d}{V}J_L$$

Axial impulse: additional Drag

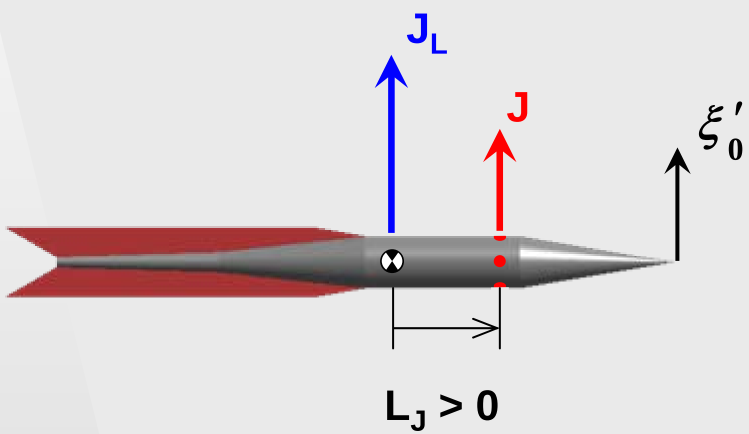
$$J_{D_2} = \int_0^\infty D_2(t)\,dt \qquad C_D = C_{D\,0} + C_{D_2}\sin^2\overline{\alpha} \qquad J_{D_2} = \frac{1}{2}\rho A V d C_{D_2}\int_0^\infty |\xi|^2 ds$$

$$\int_0^\infty |\xi|^2 ds = -\frac{K_{F_0}^2}{2\lambda_F} - \frac{K_{S_0}^2}{2\lambda_S} - \frac{2\,K_{F_0}K_{S_0}\left[(\lambda_F+\lambda_S)\cos(\phi_{F_0}-\phi_{S_0}) + (\phi'_F-\phi'_S)\sin(\phi_{F_0}-\phi_{S_0})\right]}{(\lambda_F+\lambda_S)^2 + (\phi'_F-\phi'_S)^2}$$

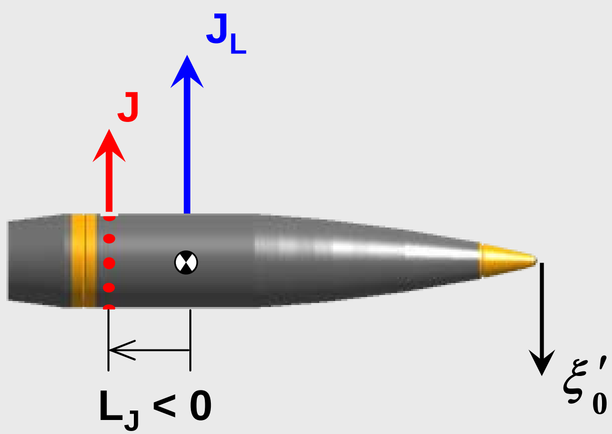
Pairing the Impulses: Location (1/2)

Goal: maximizing $J + J_L$

Fin-stabilized projectile



Spin-stabilized projectile



Rule #1: the lateral impulse must be applied
- **ahead of the center of mass for fin-stab. shells**
- **behind the center of mass for spin-stab. shells**

$$J_L = -J\, L_J \frac{C_{L\alpha}}{C_{m\alpha}} e^{i\Delta\phi}$$

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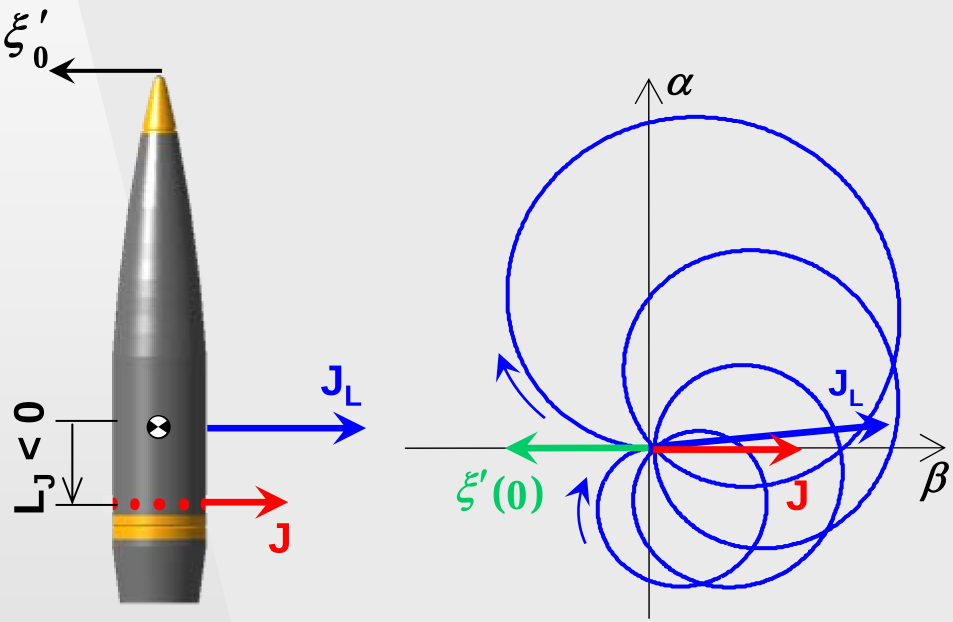
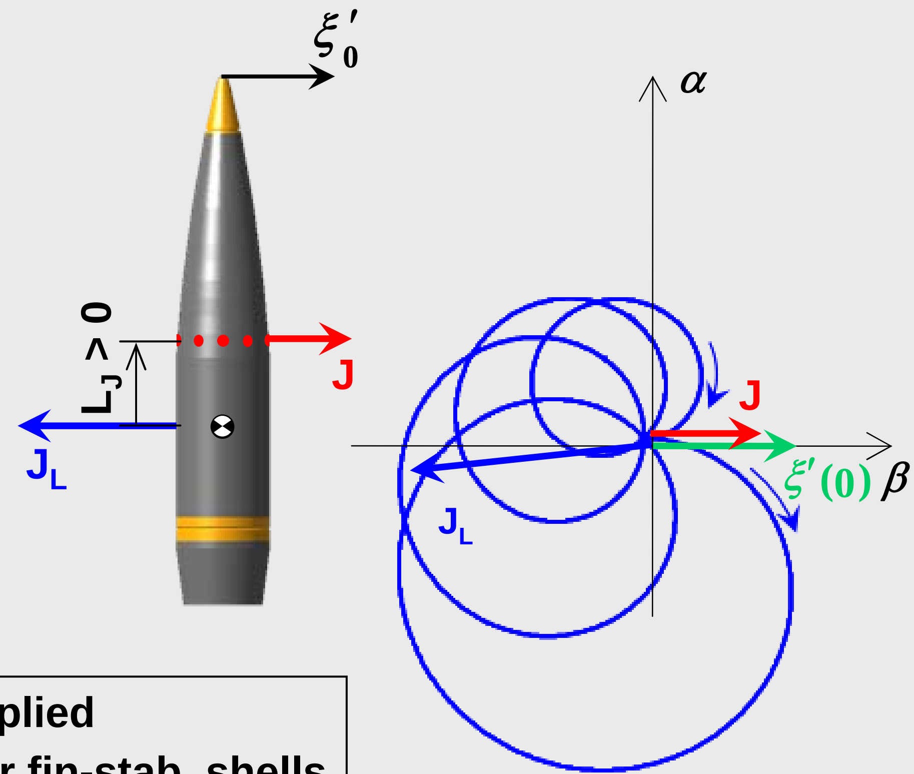
Pairing the Impulses: Location (2/2)

<u>Goal</u>: maximizing J + J_L for spin-stabilized projectile

<u>How to do it:</u>



<u>How not to do it:</u>



Rule #1: the lateral impulse must be applied
- **ahead of the center of mass for fin-stab. shells**
- **behind the center of mass for spin-stab. shells**

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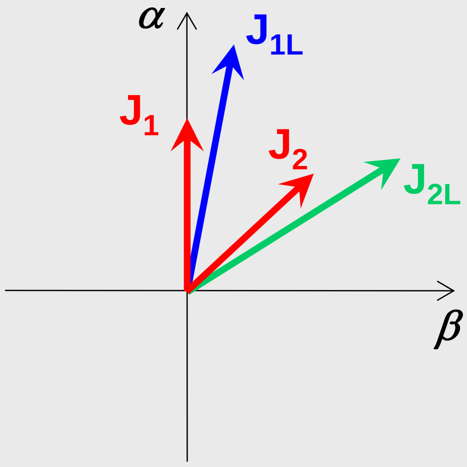
Pairing the Impulses: Orientation



<u>Total lateral impulse</u> = $(J_1 + J_{1L}) + (J_2 + J_{2L})$

- independent of $(s_2 - s_1)$
- maximum if J_1 and J_2 are aligned

Rule #2: J_1 and J_2 must be triggered at the same roll angle

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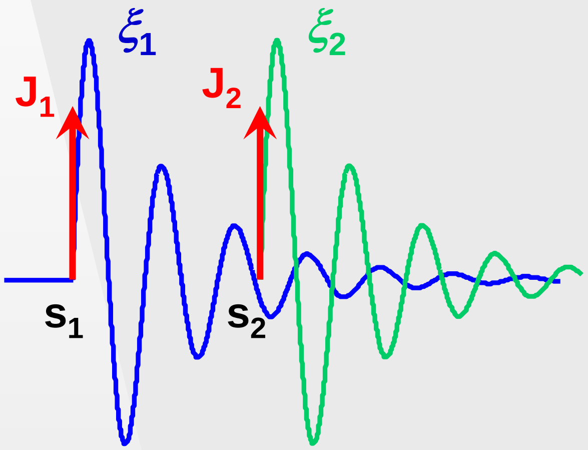
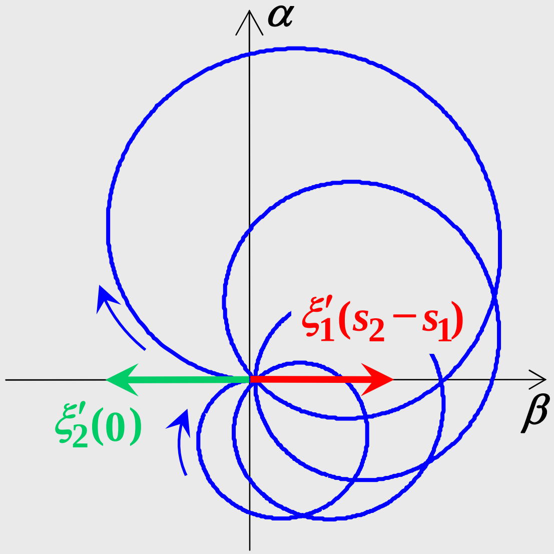
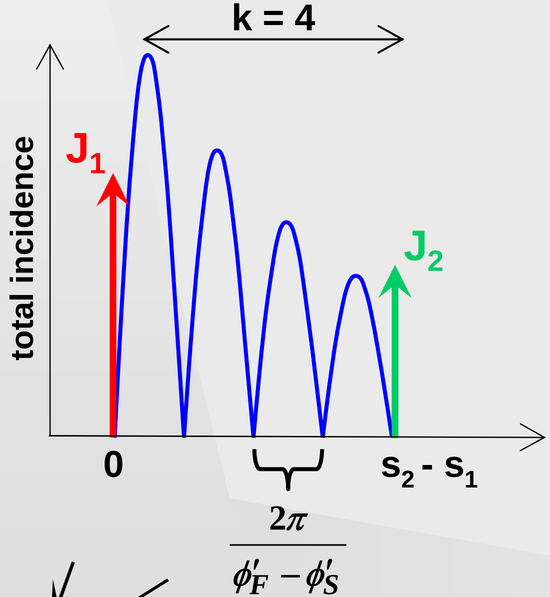
Pairing the Impulses: Timing







Linearized equation of motion:

$$\xi = \xi_1 + \xi_2$$

$$\forall s \geq s_2,\ \xi_1(s) + \xi_2(s) = 0$$

Motion strictly opposed if:

$$\xi_1(s_2 - s_1) = \xi_2(0) = 0$$

$$\xi_1'(s_2 - s_1) = -\xi_2'(0)$$

Rule #3:

$$s_2 - s_1 = k \frac{2\pi}{\phi_F' - \phi_S'}$$

$$k = \text{nearest integer to } \frac{\text{sign}(\phi_S')}{2}\left(\frac{\phi_F'}{\phi_S'} - 1\right)$$

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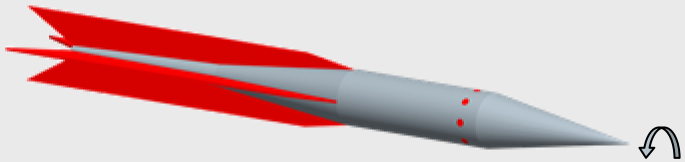
Example: GSP Shell

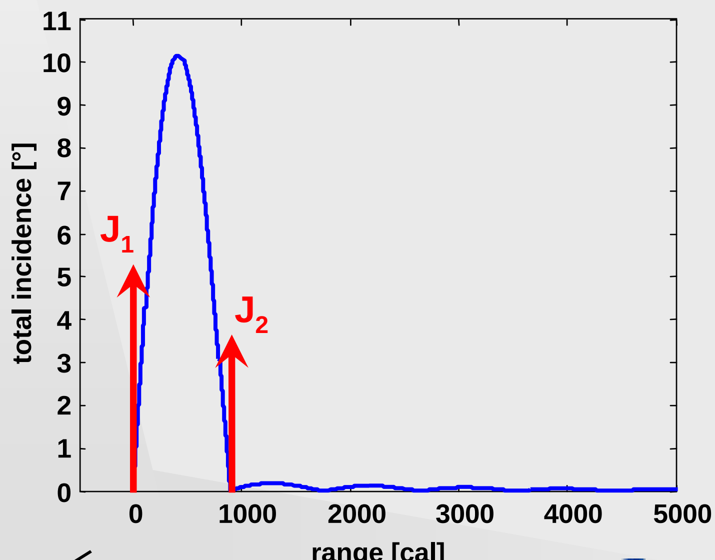
d	m	I_y	Mach	p	CD_0	CLα	CYpα	Cmα	Cmq
30 mm	0.7 kg	5.04e-3 kgm^2	3.5	22 Hz	0.19	7.6	~ 0	-5.3	~ -300

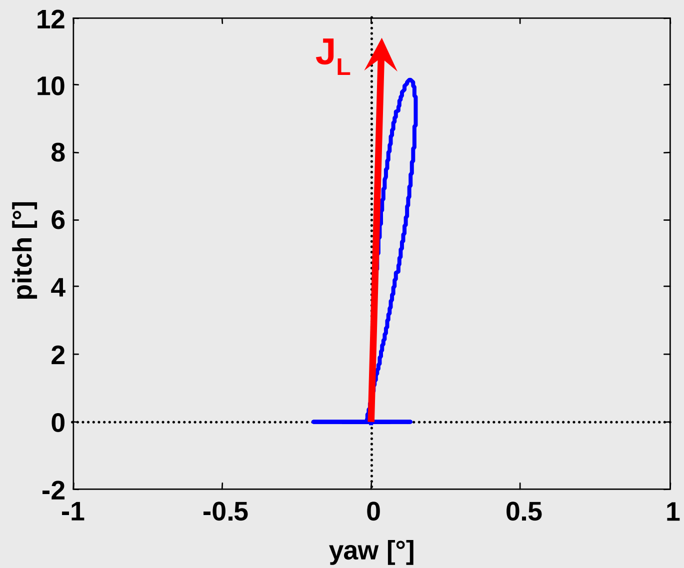
	L_J	2.5 cal
(actual)	J_1	2.0 N·s
	J_2	1.4 N·s
(26.5 m or 22.3 ms)	$s_2 - s_1$	885 cal
	$\bar{\alpha}_{max}$	10.1°

V_J	4.75 m/s
V_L	16.63 m/s
Δϕ	0.09°
V_Y	0 m/s
V_{D2}	3.98 m/s

GAIN	3.48







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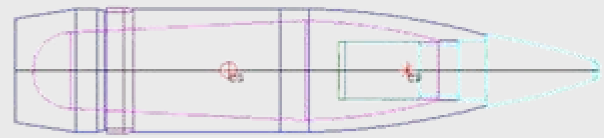
Example: 105 mm Artillery Shell (1/2)

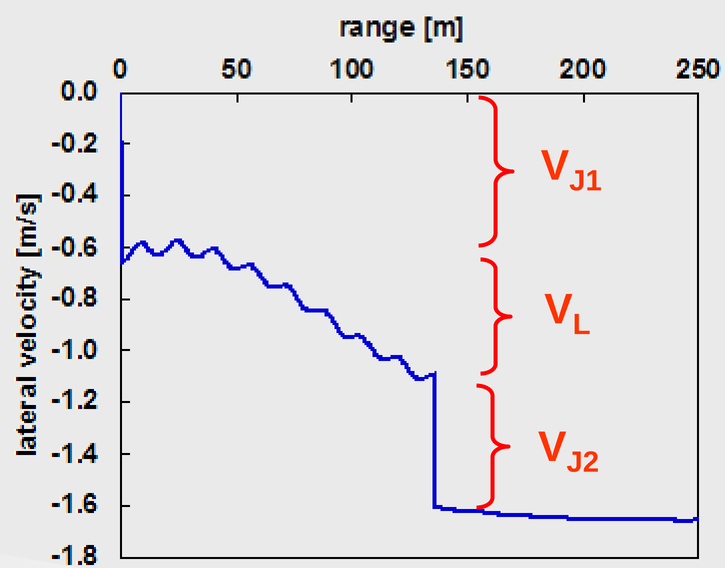
d	m	I_y	Mach	p	CD_0	$CL\alpha$	$CYp\alpha$	$Cm\alpha$	Cmq
105 mm	15.05 kg	2.19e-1 kgm^2	1.5	310 Hz	0.375	2.12	-0.8	3.6	-17



L_J	-0.51 cal
J_1	10 N·s
J_2	7.83 N·s
$s_2 - s_1$	1293 cal
$\bar{\alpha}_{max}$	1.5°

(135.6 m)

V_J	1.18 m/s
V_L	0.36 m/s
$\Delta\phi$	-3.5°
V_Y	0.05 m/s



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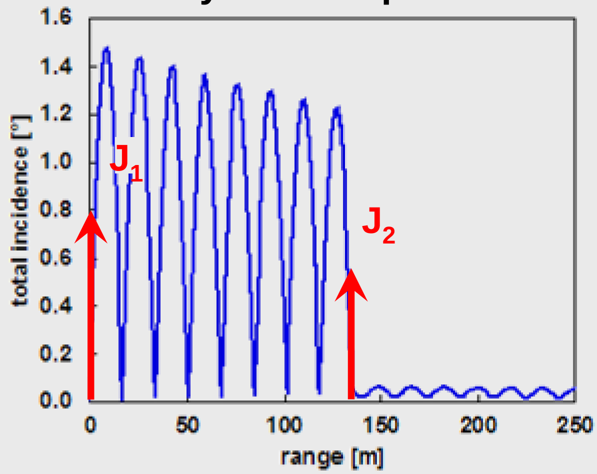
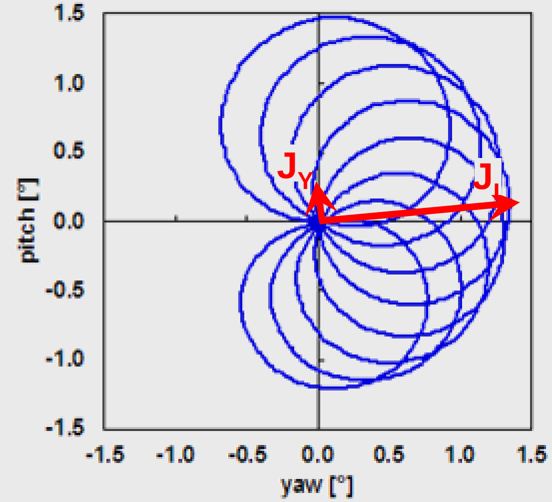
Example: 105 mm Artillery Shell (2/2)

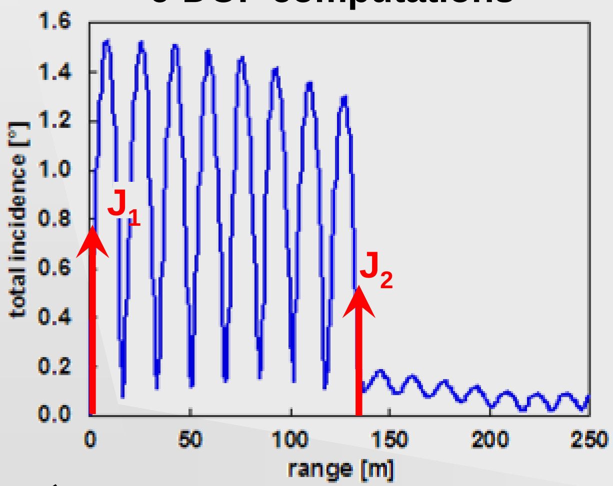
Analytical computations



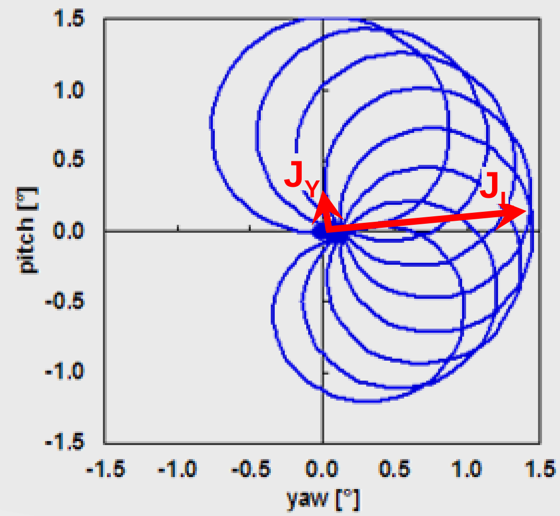
Analytical computations



6-DOF computations



6-DOF computations



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Conclusions

- An analytical model was developed to predict the angular motion of a projectile subjected to impulse thrusters
- The analytical model predicts the projectile’s angular motion very well
- A procedure to properly paired impulses in order to minimize the drag while maximizing the lateral velocity was developed
- The gain in lateral velocity obtained from the induced angular motion is significant

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