SENSOR PLACEMENT OPTIMIZATION

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Problem of Interest

- Multiple Biological detectors to be placed around and within a fixed facility as passive defense measure
- Look at sensor placement options with fast running tool to generate statistical measures
- Definition of performance metric
  - Prior work accepted “at least one hit” on sensor as adequate
  - Relationship between metric and operational use of multiple sensors
  - Consider imperfect attacks
- Overall goal to create optimization tool to determine geometry, spacing and number of sensors
Theoretical approach

- Buffon’s Needle: What is the probability that a needle hits crack in floor? It is a function of needle length and space between cracks.

\[
P (l; \alpha, b) = 1 - \frac{\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} F (\phi) \, d\phi}{\pi \alpha b},
\]

\[
F (\phi) = \alpha b - b \ell \cos \phi - \ell \alpha |\sin \phi| + \frac{1}{2} \ell^2 |\sin (2 \phi)|
\]

\[
P (l; \alpha, b) = \frac{2 \ell (\alpha + b)}{\pi \alpha b} - \ell^2.
\]
If the plane is instead tiled with congruent triangles with sides $a$, $b$, $c$ and a needle with length $l$, less than the shortest altitude is thrown, the probability that the needle is contained entirely within one of the triangles is given by

$$P = 1 + \frac{(A \alpha^2 + B \beta^2 + C \gamma^2) \ell^2}{8 \pi K^2} - \frac{(4 \alpha + 4 \beta + 4 \gamma - 3 \ell) \ell}{4 \pi K},$$

where $A$, $B$ and $C$ are the angles opposite $a$, $b$ and $c$ respectively, and $K$ is the area of the triangle.

What about dropping triangles on points, like a deadly plume on a sensor field? Too difficult – try a simulation.
Example Configuration

- CB Sensor
- Defended Region Placement
- "Plume" Contour
- Source Origin
- Placement "Margin"
- Source Region
Basic Scoring Approaches

- Count number of detections
  - Score = number of detections
  - Problems: unbounded, had to compare different size arrays; sensitivity
- One or more hits is good (war posture, false alarms not considered)
  - Score = number of runs with one or more hits / total number of runs
- More than one is better (homeland posture, avoid false alarms)
  - Score = number of runs with two or more – number of runs with zero hits / total runs
- Areas weights =>> score * plume area / base area
  - Values cases where plume covers center of defended region
- Power law weights (optimization routine, declining return)
  - Score = (2^{i-1})/2^{(i-1)} or \{0, 1, 1.5, 1.75, .. => 2.0\}
  - Allows additional weight (discrimination) for more hits
comparison of scoring approaches

- multi - zero
- multi - zero with area
- weighted area
- multi hits
Grid Configurations

Perimeter
Perimeter with Margin
Uniform Array
Dice 5
Perimeter with Center
Perimeter - 2 Tiers
Random
Circle
Circle, Margin, Center, Corners
Ellipse

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Scenario Parameters

1. Defended Region: 16 km x 19 km
2. Plume Source Region: 24 km x 27 km, centered on Defended Region
3. Plume: 25 km length, 10 degree arc width
4. Scenario Control: 2500 trials per run, fixed seed
5. Sensor Configuration: Margin = 0.0
Single Hit Performance Metric

Number of Sensors

Score

Perimeter
Uniform
Dice 5

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Perimeter vs Uniform for multiple hits

If the sensors are far apart, it is difficult to hit two or more with Perimeter.

Uniform is preferred with limited sensors.
Observations

• Dice 5 configurations offer no advantage over uniform arrays
• Configurations that conform to defended region “work better” than configurations that don’t conform
• Perimeter geometries and uniform arrays have a crossing point as number of sensors is increased
• Scoring system must take into account tactical motivations, false alarms, forensics, etc.
• Optimization using Tabu search should be able to optimize margin, spacing and number of sensors for a given area, especially with warm start provided by this tool
Future Areas for Study

• Optimization of sensor placement
  – Spacing (wind), geometry (spiral), margin, number, cost, performance
  – More realistic sensor performance/ Mixed sensitivity
• Chemical versus Bio plume size consideration
  – Topology, terrain, day/night, etc.
• Quantitative specification of perimeter/uniform cross-over point
• Non-rectangular defended regions