OUTLINE

• Background
• Problem
• Previous Work
• Proposed Solutions
BACKGROUND

- Atmospheric Transport and Dispersion (ATD) Models
- Puff or Plume Models
- Estimate Location and Population Affected

- Key Elements
  - Size and Location of Release
  - Meteorological Data
PROBLEM

• Accidental or Terrorist Release
  – Source is Unknown (Size and Location)

• Therefore Hazard Prediction is Poor

• Identification of Source is Critical
PREVIOUS WORK

• Fundamental Question for Environmental Science
  – Pollution Source Attribution

• Accidental Release
  – Chemical or Nuclear Plant or Transportation Accident

• Solution
  – Large Sensor Grid
  – Use ATD and Limited Sensor Data
PROPOSED SOLUTIONS

• FORWARD
  – Guess Source
  – Use ATD to Estimate the Hazard
  – Does it Match?
  – Iterate Guess and Recalculate
  – Lots of Runs Required

• BACKWARD
  – Reverse Time
  – Use Sensor Data and Run ATD Backward
  – NOT THAT SIMPLE
BACKWARD METHODS

- Adjoint Transport
- Reverse Diffusion
ADJOINT TRANSPORT

- Concept of Reverse Diffusion is based on the Adjoint Model
- Adjoint provides “inverse” relation between model input (release parameters) and output (sensor measurement)
- Inverse applies for a general class of sensors
ADJOINT TRANSPORT OPERATOR

• For the advection-diffusion equation

\[ L(c) = \frac{\partial c}{\partial t} + \frac{\partial}{\partial x_i} (u_i c) - \kappa \nabla^2 c \]

we have

\[ L^*(c^*) = -\frac{\partial c^*}{\partial t} - u_i \frac{\partial c^*}{\partial x_i} - \kappa \nabla^2 c^* \]

which can be interpreted as reverse time, reverse velocity, but positive diffusion
GENERAL SENSOR

- KEY TO THE METHOD – Mapping Sensor Response Back to a Source
- Assume sensor output can be expressed as a linear function (weighted integral) of the concentration field

\[ S = \int c(x, t) R^*(x, t) d^3x dt = \langle c, R^* \rangle \]

where \( R^* \) is the sensor response function
- example is point sensor at \( x_0, t_0 \) : \( R^* = \delta(x - x_0, t - t_0) \)

- Solve Adjoint System: \( L^*(c^*) = R^*(x, t) \)

- Adjoint Concentration gives Relationship between Source and Sensor Output
ILLUSTRATED SCHEMATICALLY

Source $S(x,t)$ \hspace{2cm} Forward Dispersion model $F$ \hspace{2cm} Concentration $c(x,t)$

Adjoint conc $c^*(x,t)$ \hspace{2cm} Sensor data, $d$ \hspace{2cm} Sensor $m(x,t)$

Adjoint Dispersion model $B$
MULTIPLE SENSORS

• Any sensor data, S, defines a complete field (upwind space and previous time) of release mass, Q, "possibilities"

• Use multiple sensors to determine locations of consistency, i.e., same release mass for all sensors

• Note this requires a separate reverse calculation for each sensor measurement

• Release location function
  – need a measure of the range of estimates
  – wider range implies less likely as a release estimate
**IDEALIZED CASE**

- Use HPAC to generate sensor data for a 4100kg release over 1 hour using ETEX meteorology
RELEASE ESTIMATE

- Release Location Function without/with Null data
  - release mass estimate is 2400kg
TANGENT LINEAR ADJOINT

- Utilizes Automated Differentiation
- Uses Cost Function, $J$
- Jacobian of Transport Function
  \[
  \frac{\partial J(y(x))}{\partial x_i} = \sum_k \frac{\partial F_k}{\partial x_i} \frac{\partial J}{\partial y_k}
  \]
  or \[\n  \nabla_x J(y) = K^T \nabla_y J(y) \]
- Refines Initial Source Estimate
- Refines Hazard Prediction
FUTURE WORK

• Generalize sensor and release types

• Investigate probabilistic aspects
  – can we reverse the fluctuation variance equation?

• Investigate Bayesian or other techniques for combining sensor and other data
  – improved definition of source location/strength probabilities
  – multiple source possibility
SUMMARY

• Will Provide Statistical Estimate of Source
  – Location and Strength
  – Improved Estimate of Effective Area and Population

• Proven Science that will provide an Answer