MONOTONE MEASURE THEORY AS A METHOD FOR COMBINING EVIDENCE IN THREAT SCENARIOS

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Purpose

- Pose a simple problem involving two uncertainties:
  - the uncertainty in the assignment of an event to two or more possible sets.
  - the uncertainty found in the boundary (description) of the possible sets.

- Present an approach for accounting for both uncertainties in a CB model in a natural manner.

- Demonstrate the proposed approach in an example where a Chemical/Biological weapon attack has occurred and the likelihood of casualties resulting from the attack is needed.
A simple decision support system (DSS) modeling casualties resulting from a chem/bio attack.
Problem: source data contain two uncertainties for wind flow

- Suppose both uncertainties exist in the source information for the wind flow.
  - The knowledge base for wind flow consists of approximate linguistic sets (with boundary uncertainty).
  - The wind flow at the base that is attacked is “x” and has a degree evidence in each set of the knowledge base (assignment uncertainty).
Object of this study:

To account for all source information in the DSS model, i.e. both types of uncertainty: boundary (fuzziness) and assignment (ambiguity) uncertainty.
Types of sets

Crisp Sets

\[ E \]

\[ \bullet \]

Each square of the grid represents the boundary of a set describing the event.

In the fuzzy set, \( E \) is only partially described by the set.

Only assignment uncertainty

Fuzzy Sets

\[ E \]

\[ \bullet \]

Boundary uncertainty
Types of sets

Crisp Sets

**No Boundary Uncertainty**

*“Crisp Set”*

- The box represents the set describing the event.
- The boundary of the set is well defined and understood.
- The elements are either members of the set A or not, membership in the set is binary, or equal to 1 or 0.

Fuzzy Sets

**Boundary Uncertainty** *“fuzzy set”*

- The fuzzy box represents the set containing the event.
- The boundary of the set is vague or fuzzy; not clear like “tall” or “heavy”.
- The elements can have partial membership in a set; membership varies on the interval from 0 to 1.
The importance of this study

1. Both assignment and boundary uncertainty should adequately be accounted for in a DSS.

2. Previous approaches do not adequately account for both uncertainties or are not applicable here.
Proposed Approach

- Input: the input events $x$ and a frame of discernment (knowledge base) $X$. Membership functions for the sets of $X$ and the degree of evidence for $x$ in the sets.

\[ \tilde{\mathcal{B}}, \tilde{\mathcal{C}} \subseteq X \]

*Membership functions are used to obtain the membership value for event (to be shown).

Degree of evidence for input event $x_i$ is a particular set of $X$.

Degree of evidence

\[ m_{\tilde{\mathcal{B}}}(x_i) \]

\[ m_{\tilde{\mathcal{C}}}(x_i) \]
Proposed approach

- Step 1, obtain membership value from membership function for the event value, i.e. wind flow.

- Event $= x_i$

$$\mu_{\overline{B}}(x_i) \quad \text{Membership values in sets}$$

$$\mu_{\overline{C}}(x_i)$$
Step 2, Obtain percentage of the fuzzy set represented by the degree of membership in the degrees of evidence.

\[ \eta_{\tilde{B}} = m_{\tilde{B}}(x_i) \times \mu_{\tilde{B}}(x_i) \]

\[ \eta_{\tilde{C}} = m_{\tilde{C}}(x_i) \times \mu_{\tilde{C}}(x_i) \]
Preliminary Approach

- **Step 3**, Normalize the degrees of evidence to obtain updated degree of evidence.

\[
m_{\tilde{B}}(x_i) = \frac{\eta_{\tilde{B}}}{\eta_{\tilde{C}} + \eta_{\tilde{B}}} \]

\[
m_{\tilde{C}}(x_i) = \frac{\eta_{\tilde{C}}}{\eta_{\tilde{C}} + \eta_{\tilde{B}}} \]
Satisfaction of monotone measures

- Satisfies two conditions essential for monotone measures.

\[ m(\emptyset) = 0 \]

\[ \sum_{A \in P(X)} m(A) = 1 \]

where \( P(X) \) is the set that includes all subsets of the frame of discernment, \( X \), i.e. all subsets of the power-set.
An attack has occurred

- The likelihood for casualties resulting from a chemical or biological attack that has occurred in close proximity to a military base can be inferred from the available evidence for the sets of the input events.

- Each event can be assigned to the sets that describe the event with an associated amount of evidence through expert elicitation. Base preparedness is described by two crisp sets: “Unprepared” and “Prepared”. Wind flow is described by fuzzy sets, “Directly towards base”, “Near base vicinity”, and “directly away from base.”

- The degree of evidence for the outcome sets is inferred with a rule base developed by experts.
Sets for input events

Events and the sets that describe events

<table>
<thead>
<tr>
<th>Event</th>
<th>Sets describing event</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Base preparedness)</td>
<td>“Base prepared” Y</td>
</tr>
<tr>
<td></td>
<td>“Base unprepared” N</td>
</tr>
<tr>
<td>(Wind flow direction)</td>
<td>“Directly towards base”  Ā</td>
</tr>
<tr>
<td></td>
<td>“Directly away from base”  Ĉ</td>
</tr>
<tr>
<td></td>
<td>“Flow near base vicinity”  Ĉ̅</td>
</tr>
<tr>
<td>(Casualties resulting from attack)</td>
<td>“No casualties” O₁</td>
</tr>
<tr>
<td></td>
<td>“Few casualties” O₂</td>
</tr>
<tr>
<td></td>
<td>“Moderate casualties” O₃</td>
</tr>
<tr>
<td></td>
<td>“Heavy casualties” O₄</td>
</tr>
</tbody>
</table>
Rule base from experts

Rule base used to infer the casualty likelihood

<table>
<thead>
<tr>
<th>Wind flow</th>
<th>Base Preparedness</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>N</td>
</tr>
<tr>
<td>B</td>
<td>O₁</td>
</tr>
<tr>
<td>C</td>
<td>O₁</td>
</tr>
</tbody>
</table>

Note, there are four possible outputs, O₁, O₂, O₃, and O₄ which correspond to “no”, “few”, “moderate”, and “high” casualties, respectively.
Membership functions for wind flow

Membership functions for casualties, showing the degree of membership value for \( x \) casualties. The uncertainty in the boundary is portrayed in the gradual transition of membership.
Source information for base preparedness and wind flow

Evidence assignment for base preparedness

<table>
<thead>
<tr>
<th>Base Preparedness</th>
<th>Degree of evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>( m_Y = 0.822 )</td>
</tr>
<tr>
<td>N</td>
<td>( m_N = 0.178 )</td>
</tr>
</tbody>
</table>

Evidence assignment for a specific wind flow, \( x \)

<table>
<thead>
<tr>
<th>Wind Flow</th>
<th>Degree of evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set</td>
<td></td>
</tr>
<tr>
<td>( \tilde{A} )</td>
<td>( m_{\tilde{A}} = 0.7 )</td>
</tr>
<tr>
<td>( \tilde{B} )</td>
<td>( m_{\tilde{B}} = 0.8 )</td>
</tr>
</tbody>
</table>

Note, the membership in the third fuzzy set for wind, i.e. for “flow away from base” is zero, as can be seen in the previous graph of membership functions.
Problem: fusing both boundary uncertainty and assignment uncertainty for wind flow

- Applying the fusing approach presented earlier, the boundary uncertainty can be accounted for in the evidence of wind flow.

- Our approach results in fused degrees of evidence for wind flow of:
  \[ m_{\tilde{A}} = 0.4375 \]
  \[ m_{\tilde{B}_1} = 0.5625 \]
The resulting assignment of evidence for the solution (using an inference method)

\[
\begin{align*}
    m(O_2) &= m_{11} \land m_{21} = \min(0.4375, 0.822) = 0.4375 \\
    m(O_1) &= m_{12} \land m_{21} = \min(0.5625, 0.822) = 0.5625 \\
    m(O_4) &= m_{11} \land m_{22} = \min(0.4375, 0.178) = 0.178 \\
    m(O_3) &= m_{12} \land m_{22} = \min(0.5625, 0.178) = 0.178
\end{align*}
\]

Therefore, a chem/bio weapon attack on this particular base has a likelihood in the set of no casualties of 0.5625, in the set of few casualties of 0.4375 and in the sets of moderate and high casualties of 0.178 each.
Conclusions

- Approach extends the traditional separate approaches of inferring an assignment of evidence with crisp sets to include fuzzy sets.

- The approach was demonstrated with a simple example of a terrorist attack on a military base using a chem/bio weapon. This can be extended to a more complicated terrorist attack.