MONOTONE MEASURE THEORY AS A METHOD FOR COMBINING EVIDENCE IN THREAT SCENARIOS

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Purpose

- Pose a simple problem involving two uncertainties:
  - the uncertainty in the assignment of an event to two or more possible sets.
  - the uncertainty found in the boundary (description) of the possible sets.
- Present an approach for accounting for both uncertainties in a CB model in a natural manner.
- Demonstrate the proposed approach in an example where a Chemical/Biological weapon attack has occurred and the likelihood of casualties resulting from the attack is needed.

A simple decision support system (DSS) modeling casualties resulting from a chem/bio attack.

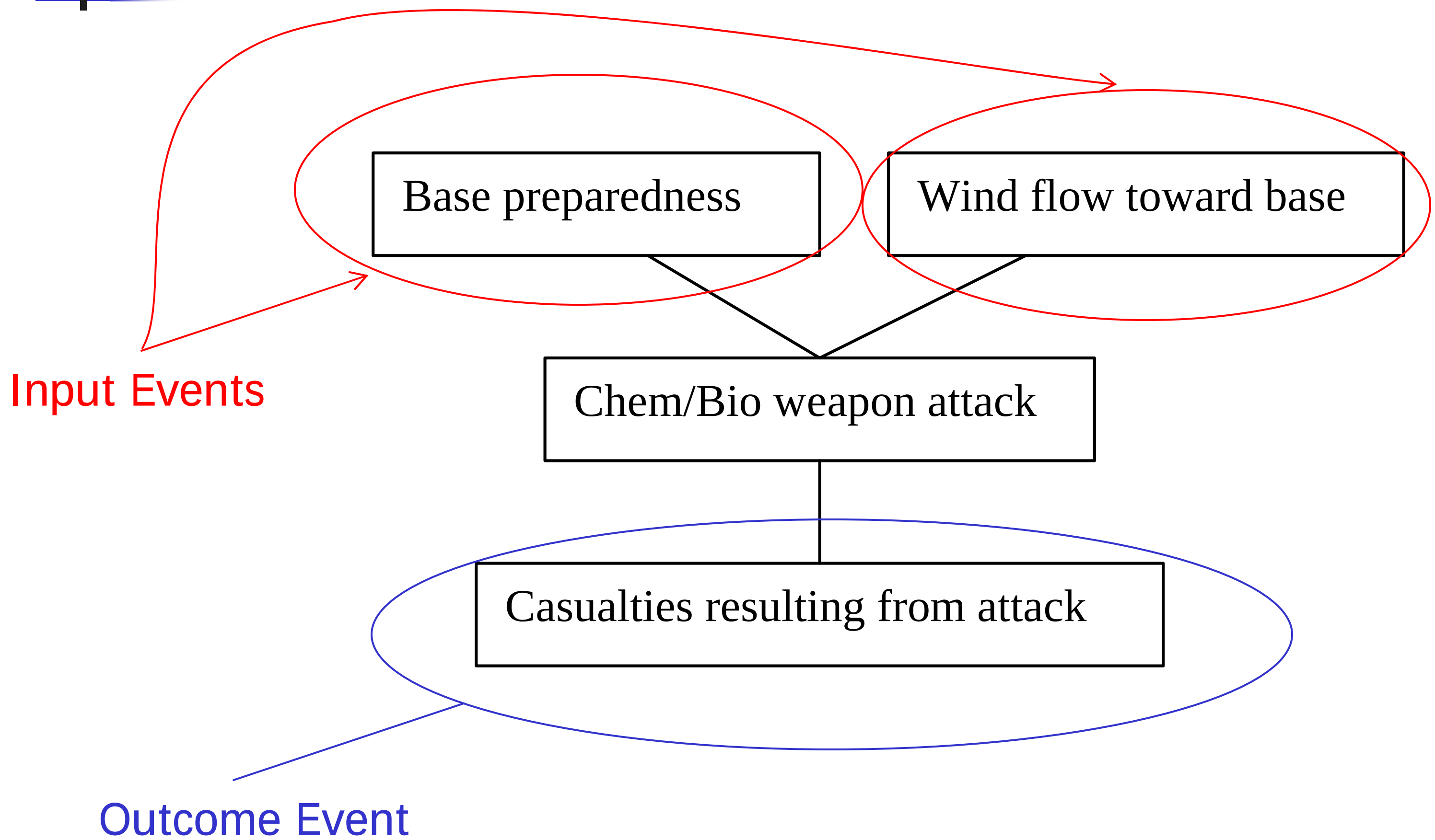


Figure 1

Problem: source data contain two uncertainties for wind flow

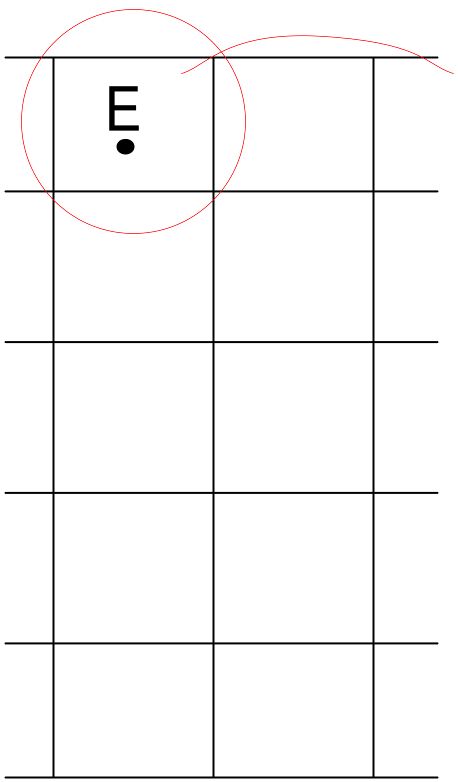
- Suppose both uncertainties exist in the source information for the wind flow.
  - The knowledge base for wind flow consists of approximate linguistic sets (with boundary uncertainty).
  - The wind flow at the base that is attacked is "x" and has a degree evidence in each set of the knowledge base (assignment uncertainty).

Object of this study:

To account for all source information in the DSS model, i.e. both types of uncertainty: boundary (fuzziness) and assignment (ambiguity) uncertainty.

Types of sets

Crisp Sets



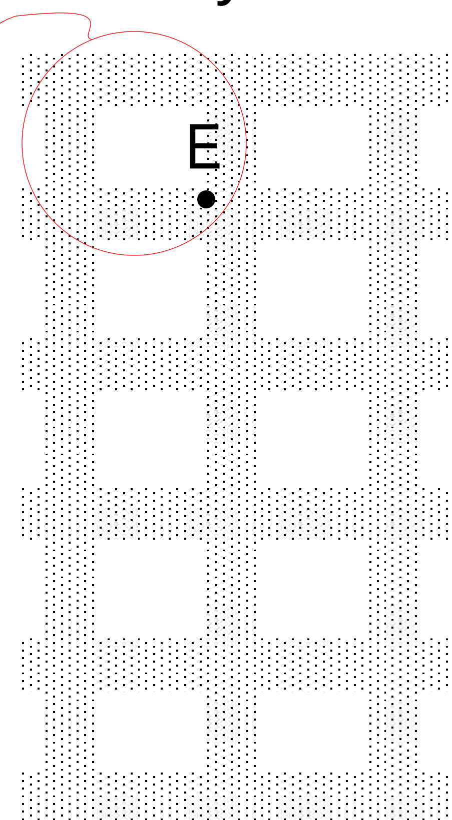
Only assignment uncertainty

E *is an event*

Each square of the grid represents boundary of a set describing the event.

In the fuzzy set **E** *is only partially described by the set.*

Fuzzy Sets



Boundary uncertainty

Types of sets

Crisp Sets

No Boundary Uncertainty ***"Crisp Set"***

•The box represents the set describing the event.

•The boundary of the set is well defined and understood.

•The elements are either members of the set A or not, membership in the set is binary, or equal to 1 or 0.

Fuzzy Sets

Boundary Uncertainty ***"fuzzy set"***

•The fuzzy box represents the set containing the event.

•The boundary of the set is vague or fuzzy; not clear like "tall" or "heavy".

•The elements can have partial membership in a set; membership varies on the interval from 0 to 1.

The importance of this study

1. Both assignment and boundary uncertainty should adequately be accounted for in a DSS.

2. Previous approaches do not adequately account for both uncertainties or are not applicable here.

Proposed Approach

- Input: the input events x and a frame of discernment (knowledge base) **X**. Membership functions for the sets of **X** and the degree of evidence for x in the sets.

$$\tilde{B}, \tilde{C} \subset \mathbf{X}$$

*Membership functions are used to obtain the membership value for event (to be shown).

Degree of evidence for input event x_i is a particular set of **X**.

Degree of evidence

$$m_{\tilde{B}}(x_i)$$

$$m_{\tilde{C}}(x_i)$$

Proposed approach

- *Step 1, obtain membership value from membership function for the event value, i.e. wind flow.*

- Event = x_i

$$\mu_{\tilde{B}}(x_i) \qquad \text{Membership values in sets}$$
$$\mu_{\tilde{C}}(x_i)$$

Preliminary Approach

- *Step 2, Obtain percentage of the fuzzy set represented by the degree of membership in the degrees of evidence.*

$$\eta_{\tilde{B}} = m_{\tilde{B}}(x_i) * \mu_{\tilde{B}}(x_i)$$

$$\eta_{\tilde{C}} = m_{\tilde{C}}(x_i) * \mu_{\tilde{C}}(x_i)$$

Preliminary Approach

- *Step 3, Normalize the degrees of evidence to obtain updated degree of evidence.*

$$\boldsymbol{m}_{\tilde{B}}(x_i) = \frac{\eta_{\tilde{B}}}{\eta_{\tilde{C}} + \eta_{\tilde{B}}}$$

$$\boldsymbol{m}_{\tilde{C}}(x_i) = \frac{\eta_{\tilde{C}}}{\eta_{\tilde{C}} + \eta_{\tilde{B}}}$$

Satisfaction of monotone measures

- Satisfies two conditions essential for monotone measures.

$$m(\varnothing) = 0$$

$$\sum_{A \in P(X)} m(A) = 1$$

where P(X) is the set that includes all subsets of the frame of discernment, X, i.e. all subsets of the power-set.

An attack has occurred

- The likelihood for casualties resulting from a chemical or biological attack that has occurred in close proximity to a military base can be inferred from the available evidence for the sets of the input events.

- Each event can be assigned to the sets that describe the event with an associated amount of evidence through expert elicitation. Base preparedness is described by two crisp sets: "Unprepared" and "Prepared". Wind flow is described by fuzzy sets, "Directly towards base", "Near base vicinity", and "directly away from base."

- The degree of evidence for the outcome sets is inferred with a rule base developed by experts.

Sets for input events

Events and the sets that describe events

<table>
<tr><th>Event</th><th colspan="2">Sets describing event</th></tr>
<tr><td>(Base preparedness)</td><td>“Base prepared” Y</td><td>“Base unprepared” N</td></tr>
<tr><td rowspan="2">(Wind flow direction)</td><td>“Directly towards base” $\tilde{A}$</td><td>“Directly away from base” $\tilde{C}$</td></tr>
<tr><td colspan="2">“Flow near base vicinity” $\tilde{B}$</td></tr>
<tr><td rowspan="3">(Casualties resulting from attack)</td><td>“No casualties” O_1</td><td>“Few casualties” O_2</td></tr>
<tr><td colspan="2">“Moderate casualties” O_3</td></tr>
<tr><td colspan="2">“Heavy casualties” O_4</td></tr>
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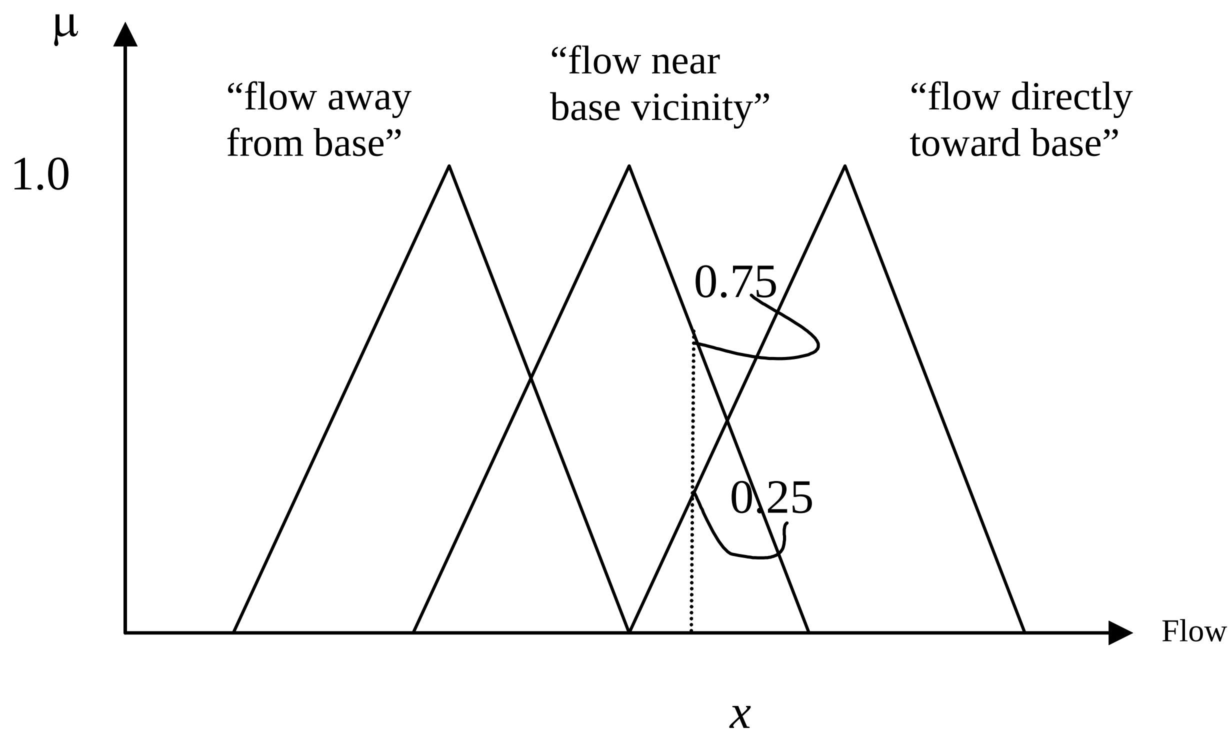
Rule base from experts

Rule base used to infer the casualty likelihood

	Base Preparedness	
Wind flow	Y	N
$\tilde{A}$	O_2	O_4
$\tilde{B}$	O_1	O_3
$\tilde{C}$	O_1	O_1

Note, there are four possible outputs, O_1, O_2, O_3, and O_4 which correspond to "no", "few", "moderate", and "high" casualties, respectively.

Membership functions for wind flow



Membership functions for casualties, showing the degree of membership value for *x* casualties. The uncertainty in the boundary is portrayed in the gradual transition of membership

Source information for base preparedness and wind flow

Evidence assignment for base preparedness

Base Preparedness	
Set	Degree of evidence
Y	$m_Y = 0.822$
N	$m_N = 0.178$

Evidence assignment for a specific wind flow, x

Wind Flow	
Set	Degree of evidence
$\tilde{A}$	$m_{\tilde{A}} = 0.7$
$\tilde{B}$	$m_{\tilde{B}} = 0.8$

Note, the membership in the third fuzzy set for wind, i.e. for "flow away from base"
Is zero, as can be seen in the previous graph of membership functions

Problem: fusing both boundary uncertainty and assignment uncertainty for wind flow

- Applying the fusing approach presented earlier, the boundary uncertainty can be accounted for in the evidence of wind flow.
- Our approach results in fused degrees of evidence for wind flow of:

$$m_{\tilde{A}} = 0.4375$$

$$m_{\tilde{B}_1} = 0.5625$$

The resulting assignment of evidence for the solution (using an inference method)

$$m(O_2) = m_{11} \wedge m_{21} = \min(0.4375, 0.822) = 0.4375$$
$$m(O_1) = m_{12} \wedge m_{21} = \min(0.5625, 0.822) = 0.5625$$
$$m(O_4) = m_{11} \wedge m_{22} = \min(0.4375, 0.178) = 0.178$$
$$m(O_3) = m_{12} \wedge m_{22} = \min(0.5625, 0.178) = 0.178$$

Therefore, a chem/bio weapon attack on this particular base has a likelihood in the set of *no casualties* of 0.5625, in the set of *few casualties* of 0.4375 and in the sets of *moderate* and *high casualties* of 0.178 each.

Conclusions

- Approach extends the traditional separate approaches of inferring an assignment of evidence with crisp sets to include fuzzy sets.
- The approach was demonstrated with a simple example of a terrorist attack on a military base using a chem/bio weapon. This can be extended to a more complicated terrorist attack.

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