Logarithms Can Be Your Friends

Controlling Peer Review Costs

November 16, 2005

Richard L. W. Welch, PhD
Chief Statistician
Northrop Grumman Corporation
Topics

- Business Objectives
- CMMI Requirements for Sub-process Control
- Why Peer Reviews?
- Data Characteristics and Difficulties
- Log-Return Model / Log-Cost Model
- The Lognormal Distribution
- Our Code Walkthrough Data on Logs
- Expanding the Capability
- Summary
Enhancing Joint STARS Capabilities

On April 8, 2005, The Process Company, LLC completed a SCAMPI Class A Appraisal in accordance with the Software Engineering Institute’s Appraisal Framework using the SE/SW CMMI® version 1.1 and determined that Northrop Grumman AGS and BMS met the goals of SEI Level 5 Process Maturity.

Andreas R. Felschow
SEI Authorized Lead Appraiser
The Process Company, LLC

Kevin M. Cotherman
SEI Authorized Lead Appraiser
The Process Company, LLC

ISO / TickIT
CMMI L5

E-8C Joint STARS

CMMI L5
CMMI Higher Levels – Differences in Behavior

At Level 3.....
- **Management Reacts**
  - Comparative Rather Than Statistical Analysis
  - Process Capability Not Understood
- Measurement Program
  - Data Available for Analysis
  - Analysis at Project Level
  - Data Quality Often Still a Concern

At Level 4.....
- **Management Anticipates**
  - Predicting Results of Critical Processes
  - Evaluating Outcomes Relative to Capability
- Measurement Program
  - Data Relied on for Decision-making
  - Data Analyzed at Organization and Project Levels

At Level 5.....
- **Management Performs “Pre-emptive Strikes”**
  - Identifying & Removing Systemic Process Issues
  - Predicting Results of Innovative Improvements
- Measurement Program
  - Data Relied on for Cost/Benefit Analysis
  - Benefits Forecasted for Technology or Process Optimization
Quantitative Management

CMMI Level 4

- Establish an Organizational Baseline and Models of Process Performance
  - Average Performance (Effort, Duration, Quality, …)
  - Range of Performance Variation
  - Contribution of Sub-process Performance to Higher Level Processes

- Manage Project To Achieve Quantitative Process Performance Goals
  - Establish Project Goals Based on Organizational Performance
  - Select Sub-processes To Quantitatively Manage
  - Demonstrate Quantitative Control
  - Identify and Correct Special Causes of Performance Variation
  - Feed Data Back to the Organization
Voice of the Process

Quantitative Sub-Process Management

- A Stable Process
  - Operates Within the Control Limits 99.7% of the Time
  - Meets Budget
  - Offers Opportunities for Systematic Process Improvement
Why Peer Reviews?

- **Ubiquity**
  - Many Work Products Reviewed Throughout Software Development Life Cycle
    - Design Artifacts
    - Code
    - Test Plan, Procedures & Reports

- **Frequency**
  - High Data Rates

- **Influence**
  - Approximately 10% of the Software Development Effort Is Spent on Peer Reviews and Inspections
  - Code Walkthroughs Represent Biggest Opportunity
Prior State

SW-CMM Level 4

- Software Development Baseline Characterized by Life Cycle Phase
  - SW Requirements-Design-Code & Verification-SW Integration-System Test
  - 10+ Year Process Improvement Record Resulted in Costs Reduced by Over 67%

- Lower Level Elements Tracked and Managed with Earned Value System

- No “Above the Shop Floor” Experience with Statistical Sub-process Control

- Issues with Peer Review Quality
  - Inconsistent Data
  - Superficial Results
Data Characteristics

Raw Data

Andersen-Darling Test $p < 0.005$

**Data Non-normality Violates Probability Model**

Summary for Cost/LOC

| Percent | 99.9 | 99 | 95 | 90 | 80 | 70 | 60 | 50 | 40 | 30 | 20 | 10 | 5 | 0.1 |

Probability Plot of Cost/LOC

Normal
Can Code Walkthroughs Be Controlled?

- **Difficulties**
  - 11% False Alarm Rate (Chebyshev’s Inequality)
    - **Penalizes Due Diligence in Reviewing Code**
  - No Meaningful Lower Control Limit
    - **Does Not Flag Superficial Reviews**
  - Arithmetic Mean Distorts the Central Tendency
    - **Apparent Cost Will Not Meet Budget**
<table>
<thead>
<tr>
<th>Log-Return Model</th>
<th>Log-Cost Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stock Sales</strong></td>
<td><strong>Peer Reviews</strong></td>
</tr>
<tr>
<td>Consider a stock sale in terms of the number of shares sold for a certain price</td>
<td>Consider a code walkthrough in terms of the number of lines of code reviewed in a certain number of hours</td>
</tr>
<tr>
<td>The natural logarithm of the difference between the current and the next per share sale price is normally distributed with zero mean and a constant standard deviation</td>
<td>By analogy, the natural logarithm of the difference in cost between the current and the next peer review will be normally distributed with zero mean and a constant standard deviation</td>
</tr>
<tr>
<td>Cost basis</td>
<td>Cost Basis</td>
</tr>
<tr>
<td>$s per Share Stock Price</td>
<td>Hours per Line of Code Reviewed</td>
</tr>
</tbody>
</table>
Consequences

Log-Return Model
Stock Sales

- Stock prices themselves are lognormally distributed
- The natural logarithms of stock prices follow a normal distribution
- Thus, the log-return data meet the assumptions needed for successful control charting

Log-Cost Model
Peer Reviews

- Peer review costs are lognormally distributed
- The natural logarithms of the peer review costs follow a normal distribution
- Thus, the log-cost data meet the assumptions needed for successful control charting
Consider a stochastic process \( X_{-2}, X_{-1}, X_0, X_1, X_2, \ldots \) that represents an asset price recorded over time, like a daily sequence of prices for shares of a stock or other commodity.

We assume at time \( t \) that the realization \( x_t \) of \( X_t \) is known, but the realization \( x_{t+1} \) of \( X_{t+1} \) is unknown.

The single-period log-return, \( \ln(X_{t+1}/x_t) \), is random and assumed to be normally distributed, at the given time \( t \).

Under these assumptions, \( X_{t+1}/x_t \) is a lognormally distributed random variable, and therefore, so is \( X_{t+1} \).

Math Details extracted from:
http://www.riskglossary.com/articles/lognormal_distribution.htm
Salient Properties of the Model

- When log-returns are normally distributed, the corresponding prices are lognormally distributed
  - This model “is one of the most ubiquitous models in finance”
- The distribution of log-returns and share prices have been validated empirically by many market studies accessible on the web
- For short time periods in a stable market, the mean return is 0

Quotation from:
http://www.riskglossary.com/articles/lognormal_distribution.htm
Lognormal Density Function

\[ f(x) = \begin{cases} 
\exp \left( -\frac{1}{2} \left( \frac{\ln(x) - \mu}{\sigma} \right)^2 \right) & x > 0 \\
x \sigma \sqrt{2\pi} & x \leq 0
\end{cases} \]

\[ X \sim \Lambda[\mu, \sigma^2] \quad Y = \ln(X) \sim N[\mu, \sigma^2] \]

\[ E(X) = \exp(\mu + \sigma^2 / 2) \]

\[ Var(X) = (\exp(\sigma^2) - 1)\exp(2\mu + \sigma^2) \]

Math details can be found in any standard mathematical statistics reference, see for example, http://en.wikipedia.org/wiki/Lognormal_distribution.
Our Data on Logs

Andersen-Darling Test $p < 0.759$

A Textbook Demonstration
The Transformed Control Chart

- Impacts
  - False Alarms Minimized
  - Meaningful Lower Control Limit
  - Geometric Mean Preserves the Budget
    - OK, You Still Have to Find the Antilog

An In-control, Stable Process
One Year Later . . .

Expanding the Capability

- Test, SW Design

ASU Physical Model Peer Review Cost Data - 23SEP05

ASU Test Procedures Peer Review Cost Data - 23SEP05

ASU Software Threads Peer Review Cost Data - 23SEP05

ASU Task Description Peer Review Cost Data - 23SEP05
Summary

With the Log-cost Model

- **Peer Review Subprocesses Are In-control and Capable** of meeting baseline budget allocations
- **Due Diligence Is Rewarded**
- **Superficial Reviews Are Detected**
- **False Alarm Rate Reduced**
  - Greater Than $40 \times$ Improvement

**Enhanced Sub-Process Control for CMMI Levels 4 and 5**